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ON FINITE SUMMATION FORMULAE INVOLVING GENERALIZED KAMPE DE FERIET SERIES AND $\overline{H}$-FUNCTION

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Abstract

Object of the present paper is to establish a number of finite summation formulae involving the generalized Kampe de Feriet series and the $\overline{H}$-function. A few finite summation formulae involving hypergeometric polynomials are mentioned as particular cases.

Keywords: Kempe de Feriet series, $\overline{H}$-function, summation formula.

INTRODUCTION

The $\overline{H}$-function is defined and represented as follows:

$$\overline{H}[Z]=\overline{H}_{P,Q}^{M,N}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,p}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\overline{\phi}(\xi)\,z^{\xi}\,d\xi \tag{1.1}$$

Where

$$\overline{\phi}(\xi)=\frac{\prod_{j=1}^{M}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{N}\left\{\Gamma(1-a_j+\alpha_j\xi)\right\}^{A_j}}{\prod_{j=M+1}^{q}\left\{\Gamma(1-b_j-\beta_j\xi)\right\}^{B_j}\prod_{j=N+1}^{P}\Gamma(a_j-\alpha_j\xi)} \tag{1.2}$$

For the condition of existence of $\overline{H}$[z] etc., we refer to Buschman and Srivastava [1990]. These conditions shall be assumed to be satisfied by the various $\overline{H}$-function occurring in this paper.

The generalized Kempe de Feriet series is defined and represented here as follows:

$$F\left[z_1,\dots,z_r\right]=F_{q:m_1,\dots,m_r}^{p;q_1,\dots,q_r}\left[\begin{matrix}(a_p):A;\\(\alpha_q):B;\end{matrix}z_1,\dots,z_r\right]=\sum_{s_1,\dots,s_r=0}^{\infty}\Delta(s_1,\dots,s_r)\frac{z_1^{s_1}}{s_1!}\cdots\frac{z_r^{s_r}}{s_r!} \tag{1.3}$$

Where

$$\Delta(s_1,...,s_r)=\frac{\prod_{j=1}^{p}(a_j)_{s_1+...+s_r}\prod_{j=1}^{q_1}(b_j^{'})_{s_1}...\prod_{j=1}^{q_r}(b_j^{(r)})_{s_r}}{\prod_{j=1}^{q}(\alpha_j)_{s_1+...+s_r}\prod_{j=1}^{m_1}(\beta_j^{'})_{s_1}...\prod_{j=1}^{m_r}(\beta_j^{(r)})_{s_r}}$$

A and B stand for the array of parameters given below:

$$A=(b_{q_1}^{'});...;(b_{q_r}^{(r)}) \quad ; B=(\beta_{m_1}^{'});...;(\beta_{m_r}^{(r)})$$

For the conditions of existence of $F[z_1,...,z_r]$, it is referred to Srivastava and Manocha [1984] (p.65-66, Eqs. (23)-(25)). It is assumed that the various F[.] functions occurring below satisfied these conditions. For the sake of brevity: following simplifying notations will be used:

$$Z=z_1,...,z_r \ ; Q=q,...,q_r \quad M=m_1,...,m_r \, .$$

RESULTS REQUIRED

The following results, which are essentially required here:

$$\sum_{n=0}^{N}\binom{N}{n}\left(-\frac{y}{x}\right)^{n} f(K-n,n)\,F^{p:Q;1+\nu_1+\rho;1+\nu_2+h}_{q:M;\mu_1+\sigma\ ;\mu_2+k}\left[\begin{matrix}(a_p):A;-n,(C_{\nu_1}),(\gamma_\rho)+N-n;-N+n(C^{'}_{\nu_2});(\xi_h)+n\\(\alpha_q):B,(d_{\mu_1}),(\delta_\sigma)+N-n;(d^{'}_{\mu_2});(\zeta_k)+n;\end{matrix};Z,x,y\right]$$

$$=\sum_{l,m=0}^{l+m\le N}\binom{N}{l+m}(l+m)!\,\Omega(N;l,m)\frac{Y^l}{l!}\frac{(-y)^m}{m!}\,{}_{1+\sigma+h}F_{\rho+k}\left[\begin{matrix}-N+l+m,1-(\delta_\sigma)-N,(\xi_h)+l+m\\1-(\gamma_\rho)-N,(\zeta_k)+l+m\end{matrix};(-1)^{\rho+\sigma}\frac{Y}{x}\right]F^{p:Q}_{q:M}\left[\begin{matrix}(a_p)+l+m:A\\(\alpha_q)+l+m:B\end{matrix};z\right] \quad (2.1)$$

With

$$f(m,n)=\frac{\prod_{j=1}^{\rho}(\gamma_j)_m\prod_{j=1}^{h}(\xi_j)_n}{\prod_{j=1}^{\sigma}(\delta_j)_m\prod_{j=1}^{k}(\zeta_j)_n} \quad (2.2)$$

And

$$\Omega(N;l,m)=\frac{\prod_{j=1}^{\rho}(\gamma_j)_N\prod_{j=1}^{p}(a_j)_{l+m}\prod_{j=1}^{h}(\xi_j)_{l+m}\prod_{j=1}^{\nu_1}(c_j)_l\prod_{j=1}^{\nu_2}(c^{'}_j)_m}{\prod_{j=1}^{\sigma}(\delta_j)_N\prod_{j=1}^{q}(\alpha_j)_{l+m}\prod_{j=1}^{k}(\zeta_j)_{l+m}\prod_{j=1}^{\mu_1}(d_j)_l\prod_{j=1}^{\mu_2}(d^{'}_j)_m} \quad (2.3)$$

$$\sum_{n=0}^{N}\binom{N}{n}\left(-\frac{y}{x}\right)^{n} f(K-n,n)\,F^{p:Q;1+\rho;1+h}_{q:M;\sigma\ ;k}\left[\begin{matrix}(a_p):A;-n,(\gamma_\rho)+N-n;-N+n;(\xi_h)+n\\(\alpha_q):B,(\delta_\sigma)+N-n;(\zeta_k)+n;\end{matrix};Z,x,y\right]$$

$$=\frac{\prod_{j=1}^{\rho}(\gamma_j)_N}{\prod_{j=1}^{\sigma}(\delta_j)_N}\,{}_{1+\sigma+h}F_{\rho+k}\left[\begin{matrix}-N,1-(\delta_\sigma)-N;(\xi_h)\\1-(\gamma_\sigma)-N,(\zeta_k)\end{matrix};(-1)^{\rho-\sigma}\frac{y}{x}\right]F^{p:Q}_{q:M}\left[\begin{matrix}(a_p):A\\(\alpha_q):B\end{matrix};Z\right] \quad (2.4)$$

f (m, n) being given by Eq. (2.2) above.

The following integrals are also required in the sequal: with

$$U=\left(\frac{t^{\mu}}{x^{\mu}}\right)^{\rho}\left(1-\frac{t^{\mu}}{x^{\mu}}\right)^{\sigma}$$

$$\int_0^x t^{a}\left(\frac{t^{\mu}}{x^{\mu}}\right)^{\alpha}\overline{H}_{P_1,Q_1}^{M_1,N_1}[\omega U]\,dt=\frac{x^{a+1}}{\mu}\overline{H}_{P+2_1,Q_1+1}^{M_1,N_1+2}\left[\omega\left|\begin{matrix}\left(1+\frac{a}{\mu},\rho_1;1\right),(-\alpha,\sigma_1;1),A^{*}\\ B^{*},\left(-\alpha+\frac{a}{\mu},\rho_1+\sigma_1;1\right)\end{matrix}\right.\right] \quad (2.5)$$

Provided that

$$\operatorname{Re}(a)+\mu\rho\wedge+1>0,\ and\ \operatorname{Re}(\alpha)+\sigma\wedge+1>0;$$

With

$$V=\left(\frac{x^{\mu}}{t^{\mu}}\right)^{\delta}\left(1-\frac{x^{\mu}}{t^{\mu}}\right)^{t}$$

$$\int_x^{\infty} t^{a-1}\left(1-\frac{x^{\mu}}{t^{\mu}}\right)^{\alpha}\overline{H}_{P_1,Q_1}^{M_1,N_1}[\omega V]\,dt=\frac{x^{a}}{\mu}\overline{H}_{P_1+2,Q_1+1}^{M_1,N_1+2}\left[\omega\left|\begin{matrix}\left(1+\frac{a}{\mu},\rho;1\right),(-\alpha,\sigma_1;1),A^{*}\\ B^{*},\left(-\alpha+\frac{a}{\mu},\rho_1+\sigma_1;1\right)\end{matrix}\right.\right] \quad (2.6)$$

Provided that

$$\operatorname{Re}(a)+\mu\rho_1\wedge>0,\ and\ \operatorname{R}(\alpha)+\sigma_1\wedge+1>0;$$

In the above expression $\wedge=\min\limits_{1\le j\le M_1}\operatorname{Re}\left(f_j/F_j\right)$.

In what follows, following notations for the sake of brevity will be used:

$$F(Z;l,m)=F_{q:M}^{p:Q}\left[\begin{matrix}(a_p)+l+m:A\\(\alpha_q)+l+m:B\end{matrix};z\right] \quad (2.7)$$

$$\overline{H}_1[\omega;n]=\overline{H}_{P_1+2,Q_1+1}^{M_1,N_1+2}\left[\omega\left|\begin{matrix}\left(1-\frac{\eta+N}{\mu},\rho_1;1\right),(-\alpha,\sigma_1;1),A^{*}\\ B^{*},\left(-\alpha-\frac{\eta+N}{\mu},\rho_1+\sigma_1;1\right)\end{matrix}\right.\right] \quad (2.8)$$

$$A^{*}=(e_j,E_j;A_j)_{1,N_1},(e_j,E_j)_{N_1+1,P_1};B^{*}=(f_j,F_j)_{1,M_1},(f_j,F_j;B_j)_{M_1+1,Q_1}$$

THE MAIN RESULTS

The following finite summation formulae are established in this section:

$$\sum_{n=0}^{N}\binom{N}{n}f(N-n,n)\overline{H}_1[\omega;n+1]F_{q:M;\mu_1+\sigma;\mu_2+k}^{p:Q;1+\nu_1+\rho;1+\nu_2+h}\left[\begin{matrix}(a_p):A;-n,(c_{\nu_1}),(\gamma_\rho)+K-n;-N+n;(c'_{\nu_2}),(\xi_h)+n\\(\alpha_q):B;(d\mu_1),(\delta_\sigma)+K-n;(d'_{\mu_2}),(\zeta_k)+n\end{matrix};z,x,y\right]$$

$$=\sum_{l,m=0}^{l+m\le N}f(Z;l,m)\frac{Y^{l}(-y)^{m}}{l!\,m!}\left\{\sum_{n=0}^{K-l-m}\frac{(-1)^{n}K!}{n!(N-l-m-n)!}\Omega(N-n;l,m)\frac{\prod_{j=1}^{h}(\xi_j+l+m)_n}{\prod_{j=1}^{k}(\zeta_j+l+m)_n}\overline{H}_1[\omega;n+1]\left(\frac{Y}{x}\right)^{n}\right\} \quad (3.1)$$

Provided that $\mathrm{Re}(\eta)+\mu\rho_1\wedge+1>0$ *and* $\mathrm{Re}(\alpha)+\sigma_1\wedge+1>0$; $f(.),\Omega(.)$ being given by Eqs. (2.2) and (2.3).

$$\sum_{n=0}^{N}\sum_{m=0}^{N-n}\frac{N!(-1)^m}{(N-n-m)!n!}h(N-n;n,m)\overline{H}_1[\omega;m+1]\frac{\prod_{j=1}^{\nu_2}(c'_j)_m}{\prod_{j=1}^{\mu_2}(d'_j)_m}F^{p:Q:1+\nu_1+\rho}_{q:M:\mu_1+\sigma}\left[{}^{(a_p)+m:A;-n,(c\gamma_1),(\gamma_\rho)+N-n}_{(\alpha_q)+m:B;(d_{\mu_1}),(\delta_\sigma)+N-n};Z,x\right]\left(-\frac{Y}{x}\right)^n\frac{y^m}{m!}$$

$$=\sum_{l,m=0}^{l+m\le N}\frac{N!(-1)^m}{(N-l-m)!}\Omega(N;l,m)\overline{H}_1[\omega;m+1]F[Z;l,m]_{1+\sigma+h}F_{\rho+k}\left[{}^{-N+l+m,1-(\delta_\sigma)-N,(\xi_h)+l+m}_{1-(\gamma_\rho)-N,(\zeta_k)+l+m};(-1)^{\rho-\sigma}\frac{Y}{x}\right]\frac{Y^l}{l!}\frac{y^m}{m!} \quad (3.2)$$

With

$$h(u;n,m)=\frac{\prod_{j=1}^{\rho}(\gamma_j)_u\prod_{j=1}^{h}(\xi_j)_{m+n}\prod_{j=1}^{p}(a_j)_m}{\prod_{j=1}^{\sigma}(\delta_j)_u\prod_{j=1}^{h}(\zeta_j)_{m+n}\prod_{j=1}^{p}(\alpha_j)_m}$$

And $\mathrm{Re}(\eta)+\mu\rho_1\wedge+1>0$ *and* $\mathrm{Re}(\alpha)+\sigma_1\wedge+1>0$;

$$\sum_{n=0}^{N}\sum_{m=0}^{N-n}\frac{N!(-1)^{m+n}}{(N-m-n)!n!m!}h(N-n;n,m)\overline{H}_1[\omega_1;m+n+1]F^{p:Q;1+\rho}_{q:M;\sigma}\left[{}^{(a_p):A;-n,(\gamma_\rho)+N-n}_{(\alpha_q):B;(\delta_\sigma)+N-n};Z,x\right]x^{-n}y^{m+n}$$

$$=\sum_{n=0}^{K}(-K)_n\, f(K-n,n)\overline{H}_1[\omega;n+1]F^{p:Q}_{q:M}\left[{}^{(a_p):A}_{(\alpha_q):B};Z\right]\left(\frac{Y}{x}\right)^n \quad (3.3)$$

With $\mathrm{Re}(\eta)+\mu\rho_1\wedge+1>0$ *and* $\mathrm{Re}(\alpha)+\sigma_1\wedge+1>0$;

$$\sum_{n=0}^{N}\sum_{l=0}^{m}\binom{N}{n}\frac{(-n)_l}{l!}h(N-n+l;n-l,l)\overline{H}_1[\omega;n-l]\frac{\prod_{j=1}^{\nu_1}(c_j)_l}{\prod_{j=1}^{\mu_1}(d_j)_l}(-Y)^n$$

$$F^{p:A:1+\nu_2+h}_{q:M:\mu_2+k}\left[{}^{(a_p)+l:A;-N+n,(c'_{\nu_2}),(\xi_n)+n}_{(\alpha_q)+l:B;(d'_{\mu_2}),(\zeta_k)+n};Z,y\right]x^{l-n}$$

$$=\sum_{l,m=0}^{l+n\le N}F(Z;l,m)\frac{y^l}{l!}\frac{(-y)^m}{m!}\left\{\sum_{n=0}^{N-l-m}\frac{(-1)N!}{n!(N-l-m-n)!}\frac{\prod_{j=1}^{h}(\xi_h+l+m)_n}{\prod_{j=1}^{k}(\zeta_h+l+m)_n}\Omega(N-n;l,m)\overline{H}_1[\omega;n]\left(\frac{Y}{x}\right)^n\right\} \quad (3.4)$$

With $\mathrm{Re}(\eta)+\mu\rho_1\wedge+1>0$ *and* $\mathrm{Re}(\alpha)+\sigma_1\wedge+1>0$;

$$\sum_{n=0}^{N}\sum_{l=0}^{n}\binom{N}{n}(-n)_l h(N-n+l;n-l,l)\overline{H}_1[\omega;n-l](-y)^n\frac{x^{l-n}}{l!}F^{p:Q;1+h}_{q:M;k}\left[{}^{(a_p)+l:A;-N+n,(\xi_h)+n}_{(\alpha_q)+l:B,(\zeta_k)+n};Z,y\right]$$

$$=\sum_{n=0}^{K}\frac{(-N)_n}{n!}f(N-n,n)\overline{H}_1[\omega;n]F^{p:Q}_{q:M}\left[{}^{(a_p):A}_{(\alpha_q):B};Z\right]\left(\frac{Y}{x}\right)^n \quad (3.5)$$

Provided that $\mathrm{Re}(\eta)+\mu\rho_1\wedge+1>0$ *and* $\mathrm{Re}(\alpha)+\sigma_1\wedge+1>0$;

OUT LINE OF PROOFS

To establish Eq. (3.1), starting with Eq. (2.1) and replacing Y by t, multiplying both sides of (2.1) by $t^{\eta}\left(Y^{\mu}-t^{\mu}\right)^{\alpha}\,\overline{H}_{P_1,Q_1}^{M_1,N_1}[\omega U]$ (U_r Being given with Eq. (2.5)) and integrating with respect to t within limits 0 to Y and then using Eq. (2.5) to evaluate the two resulting integrals on the two sides, to arrive at Eq. (3.1). The results in Eqs. (3.2) and (3.3) are established with the help of Eqs. (2.1) and (2.4) respectively by replacing y by t therein and then proceeding on lines similar to the above.

To establish Eqs. (3.4) and (3.5), starting with Eqs. (2.1) and (2.4) respectively and first replacing x by t, multiply both sides of Eqs. (2.1) and (2.4) by $t^{-\eta-\mu\alpha-1}\left(t^{\mu}-x^{\mu}\right)^{\sigma}\,\overline{H}_{P_1,Q_1}^{M_1,N_1}[\omega V]$ (V being given with Eq. (2.6)) and integrate with respect to t within limits x to ∞ and then use Eq. (2.6) to evaluate the two resulting integrals on the two sides, to arrive at Eqs. (3.4) and (4.5) respectively.

PARTICULAR CASES

(i) If, in (3.1), we put $z_1=\ldots=z_r=0$, it yields the following:

$$\sum_{n=0}^{N}\binom{N}{n} f(N-n,n)\,\overline{H}_1[\omega;n+1]\,F^{p:1+\nu_1+\rho:1+\nu_2+h}_{q:\mu_1+\sigma:\mu_2+k}\left[\begin{matrix}(a_p):-n,(c_{\nu_1}),(\gamma_\rho)+N-n:K+n,(c'_{\nu_2}),(\xi_h)+n\\(\alpha_q):(d_{\mu_1}),(\delta_\sigma)+N-n;(d'_{\mu_2}),(\zeta_k)+n\end{matrix};x,y\right]\left(-\frac{Y}{x}\right)^{n}$$

$$=\sum_{l,m=0}^{l+m\le N}\frac{Y^{l}(-y)^{m}}{l!\,m!}F(z;l,m)\left\{\sum_{n=0}^{N-l-m}\frac{(-1)^{n}N!}{n!(N-l-m-n)!}\Omega(N-n;l,m)\frac{\prod_{j=1}^{h}(\xi_j+l+m)_n}{\prod_{j=1}^{k}(\zeta_j+l+m)_n}\overline{H}_1[\omega;n+l+1]\frac{Y^{n+l}}{l!}s^{-n}\right\}\quad(4.1)$$

valid under the conditions of Eq. (3.1).

With $M=1,N=0=P,Q=2,b_1=0,b_2=-\lambda,\beta_1=1,\beta_2=\nu$, the $\overline{H}$ -function reduces to generalized Wright-Bessel function $\overline{J}_{\lambda}^{\nu,\mu}$ ([2], p.271, (8)) and we get

$$\sum_{n=0}^{N}\binom{N}{n} f(N-n,n)\frac{\Gamma\left(\frac{\eta+n+1}{\mu}+\rho_1\xi\right)\Gamma(1+\alpha+\sigma_1\xi)}{\Gamma\left(1+\alpha+\frac{\eta+n+1}{\mu}+(\rho_1+\sigma_1)\xi\right)}\overline{J}_{\lambda}^{\mu,\nu}F^{p;1+\nu_1+\rho:1+\nu_2+h}_{q;\mu_1+\sigma:\mu_2+k}\left[\begin{matrix}(a_p):-n,(c_{\nu_1}),(\gamma_\rho)+N-n:K+n,(c'_{\nu_2}),(\xi_h)+n\\(\alpha_q):(d_{\mu_1}),(\delta_\sigma)+N-n;(d'_{\mu_2}),(\zeta_k)+n\end{matrix};x,y\right]\left(-\frac{Y}{x}\right)^{n}$$

$$=\sum_{l,m=0}^{l+m\le N}\frac{(-y)^{m}}{m!}\left\{\sum_{n=0}^{N-l-m}\frac{(-1)^{n}N!}{n!(N-l-m-n)!}\Omega(N-n;l,m)\frac{\prod_{j=1}^{h}(\xi_j+l+m)_n}{\prod_{j=1}^{k}(\zeta_j+l+m)_n}\overline{H}_{2,3}^{1,2}\left[\omega\left|\begin{matrix}\left(1-\frac{\eta+n+l+1}{\mu},\rho_1;1\right),(-\alpha,\sigma_1;1)\\(0,1),(-\lambda,\nu;\mu),\left(-\alpha-\frac{\eta+n+l+1}{\mu},\rho_1+\sigma_1;1\right)\end{matrix}\right.\right]\frac{Y^{n+1}}{l!}x^{-n}\right\}\quad(4.2)$$

Where $(1-\nu)>0,(1+\nu)\geq 0,|\arg z|<\frac{1}{2}(1-\nu-\rho-\sigma)\pi$ and the conditions given with (3.1) are satisfied.

If, in Eq. (4.1), we replace M, N, P, Q BY 1,P, P, Q+1 respectively, the $\overline{H}$ -function reduces to the Wright's generalized hypergeometric function ${}_P\overline{\psi}_Q$ ([2], p.271, (7)) and we get

$$\sum_{n=0}^{N}\binom{N}{n} f(N-n;n)\frac{\Gamma\left(\frac{\eta+n+1}{\mu}+\rho_1\xi\right)\Gamma(1+\alpha+\sigma_1\xi)}{\Gamma\left(1+\alpha+\frac{\eta+n+1}{\mu}+(\rho_1+\sigma_1)\xi\right)}\,{}_P\overline{\psi}_Q\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,P}\\(b_j,\beta_j;B_j)_{1,Q}\end{matrix}\right.\right]F^{p;1+\nu_1+\rho:1+\nu_2+h}_{q;\mu_1+\sigma:\mu_2+k}\left[\begin{matrix}(a_p):-n,(c_{\nu_1}),(\gamma_\rho)+N-n;-N+n;(c'_{\nu_2}),(\xi_h)+n\\(\alpha_q):(d_{\mu_1}),(\delta_\sigma)+N-n;(d'_{\mu_2}),(d'_{\mu_2})+n\end{matrix};x,y\right]\left(-\frac{Y}{x}\right)^n$$

$$=\sum_{l,m=0}^{l+m\leq N}\frac{(-y)^m}{m!}\left\{\sum_{n=0}^{N-l-m}\frac{(-1)^n N!}{n!(N-l-m-n)!}\Omega(N-n;l,n)\frac{\prod_{j=1}^{h}(\xi_j+l+m)_n}{\prod_{j=1}^{k}(\zeta_j+l+m)_n}\overline{H}^{1,P+2}_{P+2,Q+1}\left[\omega\left|\begin{matrix}\left(1-\frac{\eta+n+l+1}{\mu},\rho_1;1\right),(-\alpha,\sigma_1;1),(a_j,\alpha_j;A_j)_{1,P}\\(b_j,\beta_j;B_j)_{1,Q},\left(-\alpha-\frac{\eta+n+l+1}{\mu},\rho_1+\sigma_1;1\right)\end{matrix}\right.\right]\frac{Y^{n+1}}{l!}x^{-n}\right\}\quad(4.3)$$

Where

$$\sum_{j=1}^{P}\alpha_j+1-\sum_{j=1}^{Q}\beta_j\equiv T>0\ ;|\arg z|<\frac{1}{2}(T-\rho_1-\sigma_1)\pi\ ,1+\sum_{j=1}^{Q}\beta_j-\sum_{j=1}^{P}\alpha_j\geq 0$$

and the conditions given with Eq. (3.1) also satisfied.

A number of other finite summation formulae can be derived from the result (3.2) to (3.5), but these are not recorded here for lack of space.

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