

NUMERICAL SOLUTION OF FLUID FLOW AND HEAT TRANSFER IN THE FINNED DOUBLE PIPE

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Abstract: Numerical simulation of the steady, laminar, forced convection heat transfer in the finned annulus is carried out for the case of fully developed incompressible flow corresponding to thermal boundary condition of uniform heat input per unit axial length with peripherally uniform temperature at any cross section. Boundary fitted curvilinear coordinates are used to overcome the singularities, being presented by the fin tip, in the solution domain. Various heat transfer and fluid flow characteristics are investigated for a range of values of the ratio of radii of inner and outer pipes, fin height and number of fins. The results calculated are in good comparison with the literature results with considerable gain in the computational time.

Keywords: Double pipe, grid generation, heat transfer.

INTRODUCTION

Development of high performance thermal systems have enthused interest in methods to enhance heat transfer. Thus heat transfer from rough surfaces has received a great deal of attention due to many practical applications for increasing the effectiveness of heat transfer. Laminar flow heat transfer occurs in a variety of engineering problems and is of particular importance where viscous fluids are heated or cooled. Since the heat transfer in these types of fluids is generally low, there is a need for augmentation. Much work has been carried out for the internally finned circular ducts [Nandakumar and Masliyah 1975, Masliyah and Nandakumar 1976, Soliman and Feingold 1977, Soliman *et al.* 1980]. However, finned double pipe geometry needs to be extensively studied for fluid flow and heat transfer characteristics. Agrawal and Sengupta [1990] considered the heat transfer enhancement by external circular fins. Syed [1997] has investigated the heat transfer enhancement in the double pipe geometry with longitudinal fins attached on the outer surface of the inner pipe. In this geometry, fin tip presents a singularity particularly when considerable fin thickness is taken. Therefore there is a need of very fine grid near the fin tip. Syed solved momentum and energy equations on four different grid levels starting from the coarsest grid of 20x10 points in (r, θ) system to the finest one of 160x80 points. This type of refinement leads to extra computational work and is considerable burden on the computer memory resources. In the present work, we have generated a body fitted numerical grid to resolve the re-entrant corner with the help of elliptic generation system. Although with the introduction of boundary conforming curvilinear coordinates the transformed partial differential equations become more complicated in the sense of having

more terms and cross derivatives but the domain on the other hand is greatly simplified since it is transformed to a fixed rectangular region regardless of its shape. It is shown here that with the help of body fitted grid the same accuracy in results can be achieved by solving the problem on one grid level. A comprehensive comparison of results is also carried out with those of Syed [1997].

PROBLEM STATEMENT

The system considered here is that of finned double pipe heat exchanger comprising two concentric pipes with tapered longitudinal fins distributed around the outer periphery of the inner pipe. Solution has been obtained for different geometrical parameters, which are fin height, annular gap, fin thickness and number of fins under the assumption of axially uniform heat flux.

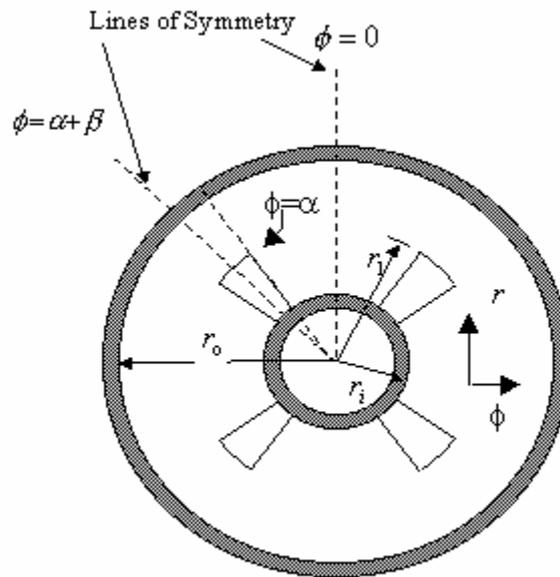


Fig. 1: Cross section of the domain.

The flow is assumed to be laminar, steady, and fully developed with viscous dissipation neglected. The fluid is considered to be Newtonian and incompressible with constant properties. All body forces are neglected. The fins are assumed to be smooth and equally spaced and have infinite conductivity, *i.e.* the fins are 100% efficient. An adiabatic thermal condition is imposed at the outer pipe. Axial conduction is neglected in fluid that is a fair assumption in our case as reported by Shah and London [1978]. A cross section of the geometry under consideration is as shown in Fig. 1. Let the outer radius of the inner pipe be denoted by r_1 and inner radius of the outer pipe be denoted by r_0 . The geometrical symmetry requires the problem to be solved in the region

where $r_i \leq r \leq r_o$ and $0 \leq \theta \leq \phi$. The dimensionless numerical domain is shown in the Fig. 2.

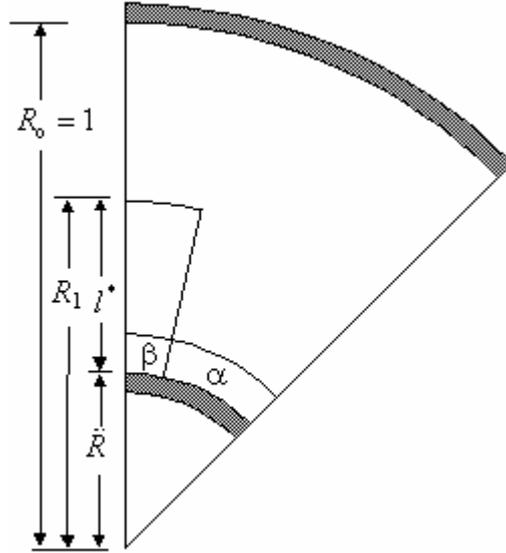


Fig. 2: Computational Domain.

GRID GENERATION

Due to the complexity of the annular domain and because of the re-entrant corner formed by the fin tip, a numerically generated body fitted coordinate system is applied to resolve the difficulties in discretizing the computational domain. The method used is suggested in [Thompson *et al.* 1984, 1999, Liseikin 1999]. Therefore, the domain transformation between the physical coordinates (r, θ) and the boundary fitted coordinates (ξ, η) is achieved by solving two coupled equations on the physical domain.

$$\left. \begin{aligned} \nabla^2 \xi &= \frac{g_{22}}{g} P \\ \nabla^2 \eta &= \frac{g_{11}}{g} Q \end{aligned} \right\} \quad (1)$$

g_{22}, g_{11} are the metric components of the metric tensor and g is the square of the jacobian of transformation. In the computational domain the grid generation Eq. (1) will become

$$\left. \begin{aligned} A_1(x_{\xi\xi} + x_\xi P) + A_2(x_{\eta\eta} + x_\eta Q) - 2A_3x_{\eta\xi} &= 0 \\ A_1(y_{\xi\xi} + y_\xi P) + A_2(y_{\eta\eta} + y_\eta Q) - 2A_3y_{\eta\xi} &= 0 \end{aligned} \right\} \quad (2)$$

where $A_1 = x_\eta^2 + y_\eta^2$, $A_2 = x_\xi^2 + y_\xi^2$, $A_3 = x_\xi x_\eta + y_\xi y_\eta$

Eqs. (2) are discretized using central differences. The resulting equations are solved using Successive Over Relaxation method (SOR) subject to the boundary conditions provided by the boundary point distribution, with automatic adjustment of the relaxation parameter ω as given by Syed *et al.* [1997]. The control functions P and Q are adjusted in such a manner that we get concentrated grid on and around the fin tip. The Jacobian of the transformation is given by

$$\sqrt{g} = (x_{\xi}y_{\eta} - x_{\eta}y_{\xi})$$

The transformation relations are given by Thompson *et al.* [1984] for conversion from Cartesian to Curvilinear coordinates. Following the footsteps, relations have been established for the conversion from polar to curvilinear coordinates. For convenience expressions for the first and second derivatives are given below:

$$u_r = \frac{1}{\sqrt{g}} (r\theta_{\eta}u_{\xi} - r\theta_{\xi}u_{\eta}), \quad \frac{1}{r}u_{\theta} = \frac{1}{\sqrt{g}} (r_{\xi}u_{\eta} - r_{\eta}u_{\xi})$$

$$u_{rr} = \frac{r^2}{g} (\theta_{\eta}^2 u_{\xi\xi} + \theta_{\xi}^2 u_{\eta\eta} - 2\theta_{\xi}\theta_{\eta}u_{\xi\eta}) + \frac{r^2}{g^{3/2}} [(\theta_{\xi\xi}\theta_{\eta}^2 - 2\theta_{\xi}\theta_{\eta}\theta_{\xi\eta} + \theta_{\eta\eta}\theta_{\xi}^2) \times$$

$$(r_{\eta}u_{\xi} - r_{\xi}u_{\eta}) + (r_{\xi\xi}\theta_{\eta}^2 - 2\theta_{\xi}\theta_{\eta}r_{\xi\eta} + r_{\eta\eta}\theta_{\xi}^2)(\theta_{\xi}u_{\eta} - \theta_{\eta}u_{\xi})]$$

$$\frac{1}{r^2}u_{\theta\theta} = \frac{1}{g} (r_{\eta}^2 u_{\xi\xi} + r_{\xi}^2 u_{\eta\eta} - 2r_{\xi}r_{\eta}u_{\xi\eta}) + \frac{r}{g^{3/2}} [(\theta_{\xi\xi}r_{\eta}^2 - 2r_{\xi}r_{\eta}\theta_{\xi\eta} + \theta_{\eta\eta}r_{\xi}^2) \times$$

$$(r_{\eta}u_{\xi} - r_{\xi}u_{\eta}) + (r_{\xi\xi}r_{\eta}^2 - 2r_{\xi}r_{\eta}r_{\xi\eta} + r_{\eta\eta}r_{\xi}^2)(\theta_{\xi}u_{\eta} - \theta_{\eta}u_{\xi})]$$

The numerical grid generated in the physical domain is shown in Fig. 3. The concentration of grid lines is quite visible near the fin tip.

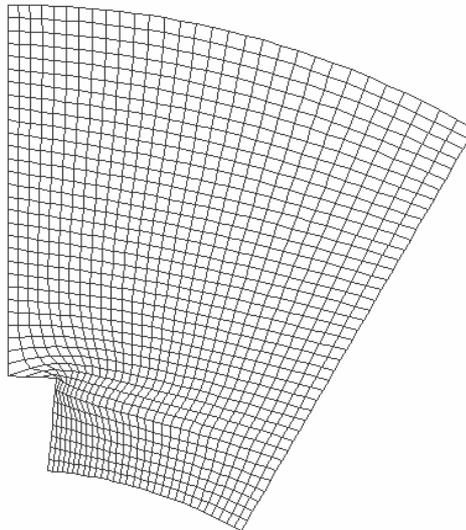


Fig. 3: Grid drawn in the computational domain.

MOMENTUM EQUATION

Under the assumptions cited earlier, the momentum equation can be written as

$$\frac{\partial^2 u'}{\partial r^2} + \frac{1}{r} \frac{\partial u'}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u'}{\partial \theta^2} = \frac{1}{\mu} \frac{dp}{dz} \quad (3)$$

where u' is the axial velocity component, p is pressure and z is the axial distance. Viscous nature of the fluid and symmetry of the numerical domain dictates the following boundary conditions:

(a) No slip conditions at the solid boundaries

$$\text{I) } u' = 0 \text{ at } r = r_i, \beta \leq \theta \leq \phi \quad \text{II) } u' = 0 \text{ at } r_i \leq r \leq r_1, \theta = \beta$$

$$\text{III) } u' = 0 \text{ at } r = r_o, 0 \leq \theta \leq \phi \quad \text{IV) } u' = 0 \text{ at } r = r_1, 0 \leq \theta \leq \beta$$

(b) Symmetry conditions

$$\frac{\partial u'}{\partial \theta} = 0 \text{ at } \theta = 0, r_i \leq r \leq r_o \quad \text{and} \quad \frac{\partial u'}{\partial \theta} = 0 \text{ at } \theta = \phi, r_i \leq r \leq r_o$$

We define the following dimensionless variables:

$$R = \frac{r}{r_o}, \quad u = \frac{u'}{u_{\max}}, \quad \hat{R} = \frac{r_i}{r_o}, \quad R_1 = \frac{r_1}{r_o}$$

$$\text{where } u_{\max} = -\frac{1}{4} \frac{dp}{dz} r_o^2 \left\{ 1 - R_m^2 + 2R_m^2 \ln R_m \right\}$$

where R_m is the dimensionless point of maximum velocity and is given by

$$R_m = \frac{r_m}{r_o} = \sqrt{\frac{1 - \hat{R}^2}{2 \ln(1/\hat{R})}}$$

In dimensionless form, the Eq. (3) becomes

$$\frac{\partial^2 u}{\partial R^2} + \frac{1}{R} \frac{\partial u}{\partial R} + \frac{1}{R^2} \frac{\partial^2 u}{\partial \theta^2} = \frac{-4}{c} \quad \text{where } c = (1 - R_m^2 + 2R_m^2 \ln R_m) \quad (4)$$

Dimensionless boundary conditions are:

(a) No slip conditions at the solid boundaries

$$u = 0 \text{ at } R = \hat{R}, \beta \leq \theta \leq \phi, \quad u = 0 \text{ at } \hat{R} \leq R \leq R_1, \theta = \beta,$$

$$u = 0 \text{ at } R = 1, 0 \leq \theta \leq \phi, \quad u = 0 \text{ at } R = R_1, 0 \leq \theta \leq \beta$$

(b) Symmetry conditions

$$\frac{\partial u}{\partial \theta} = 0 \text{ at } \theta = 0, R_1 \leq R \leq 1 \quad \text{and} \quad \frac{\partial u}{\partial \theta} = 0 \text{ at } \theta = \phi, \hat{R} \leq R \leq 1$$

Using the transformations established earlier, momentum equation and boundary conditions are transformed into the computational domain. The resulting equation after little manipulation will take the form

$$\alpha_1 u_{\xi\xi} + \alpha_2 u_{\eta\eta} - \alpha_3 u_{\xi\eta} + \alpha_4 u_{\xi} + \alpha_5 u_{\eta} = -4/c \quad (7)$$

where

$$\alpha_1 = \frac{r^2 \theta_\eta^2 + r_\eta^2}{g}, \quad \alpha_2 = \frac{r^2 \theta_\xi^2 + r_\xi^2}{g}, \quad \alpha_3 = \frac{2}{g} [r^2 \theta_\xi \theta_\eta + r_\xi r_\eta],$$

$$\alpha_4 = r_\eta \beta_5 - \theta_\eta \beta_6, \quad \alpha_5 = \theta_\xi \beta_6 - r_\xi \beta_5, \quad \beta_1 = \theta_{\xi\xi} \theta_\eta^2 - 2\theta_{\xi\eta} \theta_\eta \theta_\xi + \theta_{\eta\eta} \theta_\xi^2$$

$$\beta_2 = r_{\xi\xi} \theta_\eta^2 - 2r_{\xi\eta} \theta_\eta \theta_\xi + r_{\eta\eta} \theta_\xi^2, \quad \beta_3 = \theta_{\xi\xi} r_\eta^2 - 2\theta_{\xi\eta} r_\eta r_\xi + \theta_{\eta\eta} r_\xi^2$$

$$\beta_4 = r_{\xi\xi} r_\eta^2 - 2r_{\xi\eta} r_\eta r_\xi + r_{\eta\eta} r_\xi^2, \quad \beta_5 = \frac{r^3 \beta_1 + r \beta_3}{g^{3/2}}, \quad \beta_6 = \frac{r^3 \beta_2}{g^{3/2}} - \frac{1}{g^2} + \frac{r \beta_4}{g^{3/2}}$$

The boundary conditions are as under:

(a) No slip conditions at the solid boundaries

$$u = 0 \text{ at } \xi = 0, \quad 0 \leq \eta \leq j_\eta, \quad u = 0 \text{ at } \xi = i_\xi, \quad 0 \leq \eta \leq j_\eta$$

$$u = 0 \text{ at } \xi = i_{tip}, \quad 0 \leq \eta \leq j_\beta, \quad u = 0 \text{ at } \eta = j_\beta, \quad 0 \leq \xi \leq i_{tip}$$

(b) Symmetry conditions

$$\frac{\partial u}{\partial \eta} = \frac{R_\eta}{R_\xi} \frac{\partial u}{\partial \xi} \text{ at } \eta = 0, \quad i_{tip} \leq \xi \leq i_\xi$$

$$\frac{\partial u}{\partial \eta} = \frac{R_\eta}{R_\xi} \frac{\partial u}{\partial \xi} \text{ at } \eta = j_\eta, \quad 0 \leq \xi \leq i_\xi$$

where i_{tip} is the number of grid points in ξ direction till fin tip, i_ξ is the total number of grid points in ξ direction, j_β and j_η are the grid points on the fin tip and total number of grid points in η direction.

ENERGY EQUATION

The energy equation under the assumption of constant heat flux can be written as

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial T^2}{\partial \theta^2} + \frac{\partial^2 T}{\partial Z^2} = \frac{1}{\alpha} u' \frac{\partial T}{\partial Z},$$

where α is the thermal diffusivity and T is the temperature. The boundary conditions are

(a) At the solid boundaries

$$\text{II) } T = T_w \text{ at } r = r_i, \quad \beta \leq \theta \leq \phi \quad \text{III) } T = T_w \text{ at } r_i \leq r \leq r_1, \quad \theta = \beta$$

$$\text{IV) } \frac{\partial T}{\partial r} = 0 \text{ at } r = r_o, \quad 0 \leq \theta \leq \phi \quad \text{V) } T = T_w \text{ at } r = r_1, \quad 0 \leq \theta \leq \beta$$

(b) Symmetry conditions

$$\frac{\partial T}{\partial \theta} = 0 \text{ at } \theta = 0, \quad r_i \leq r \leq r_o \quad \text{and} \quad \frac{\partial T}{\partial \theta} = 0 \text{ at } \theta = \phi, \quad r_i \leq r \leq r_o$$

By doing the same treatment as given by Syed [1997] and using the dimensionless temperature

$$\tau = \frac{T - T_w}{\dot{Q}/k},$$

where T_w is the wall temperature, \dot{Q} is the heat transfer rate per unit axial length of the pipe and k is the thermal conductivity of the fluid, we get the dimensionless energy equation

$$\frac{\partial^2 \tau}{\partial R^2} + \frac{1}{R} \frac{\partial \tau}{\partial R} + \frac{1}{R^2} \frac{\partial^2 \tau}{\partial \theta^2} = \frac{u}{A\bar{u}} \quad (8)$$

\bar{u} is the mean velocity.

With the dimensionless boundary conditions

(a) At the solid boundaries

$$\tau = 0 \text{ at } R = \hat{R}, \beta \leq \theta \leq \phi, \quad \tau = 0 \text{ at } \hat{R} \leq R \leq R_1, \theta = \beta$$

$$\tau = 0 \text{ at } R = R_1, 0 \leq \theta \leq \beta$$

(b) Symmetry conditions

$$\frac{\partial \tau}{\partial \theta} = 0 \text{ at } \theta = 0, R_1 \leq R \leq 1$$

$$\frac{\partial \tau}{\partial \theta} = 0 \text{ at } \theta = \phi, \hat{R} \leq R \leq 1$$

$$\frac{\partial \tau}{\partial R} = 0 \text{ at } R = 1, 0 \leq \theta \leq \phi$$

In the computational domain Eq. (8) can be written as:

$$\alpha_1 \tau_{\xi\xi} + \alpha_2 \tau_{\eta\eta} - \alpha_3 \tau_{\xi\eta} + \alpha_4 \tau_{\xi} + \alpha_5 \tau_{\eta} = \frac{u}{A\bar{u}}$$

The coefficients $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ & α_5 of the above equation are same as defined in Eq. (7) and A is the area of the cross section. The transformed boundary conditions are:

(a) At the solid boundaries

$$\tau = 0 \text{ at } \xi = 0, 0 \leq \eta \leq j_{\eta}, \quad \tau = 0 \text{ at } \xi = i_{\xi}, 0 \leq \eta \leq j_{\eta}$$

$$\tau = 0 \text{ at } \xi = i_{ip}, 0 \leq \eta \leq j_{\beta}, \quad \tau = 0 \text{ at } \eta = j_{\beta}, 0 \leq \xi \leq i_{ip}$$

$$\frac{\partial \tau}{\partial \xi} = \frac{\theta_{\xi}}{\theta_{\eta}} \frac{\partial \tau}{\partial \eta} \text{ at } \xi = i_{\xi}, 0 \leq \eta \leq j_{\eta}$$

(b) Symmetry conditions

$$\frac{\partial \tau}{\partial \eta} = \frac{R_{\eta}}{R_{\xi}} \frac{\partial \tau}{\partial \xi} \text{ at } \eta = 0, i_{ip} \leq \xi \leq i_{\xi}$$

$$\frac{\partial \tau}{\partial \eta} = \frac{R_{\eta}}{R_{\xi}} \frac{\partial \tau}{\partial \xi} \text{ at } \eta = j_{\eta}, 0 \leq \xi \leq i_{\xi}$$

SOLUTION PROCEDURE

Momentum and energy equations are discretized using central differences. One-sided three point and four point difference forms are used for the first and second derivatives on the boundaries. The step size in the computational domain is designed as $\Delta\xi = 1, \Delta\eta = 1$. The resulting system of linear algebraic equations is solved using SOR method with the optimum relaxation parameter adjusted as mentioned earlier. Solution was computed for number of extreme geometries using different mesh sizes and based on these results it was decided to use a grid of 40x20 in the computational domain for all geometries as a reasonable compromise between accuracy and computer time.

RESULTS AND DISCUSSION

The range of parameters considered are, ratio of radii 0.5, half fin angle $\beta = 5^\circ$, annulus to fin height ratio $H^* = 0.2, 0.4, 0.6, 0.8, 1.0$ and number of fins $N = 6, 12, 18, 24, 30$. The values H^* correspond to 20, 40, 60 and 100% of the annulus respectively. The expressions for the bulk mean velocity \bar{u} and bulk mean temperature τ_b are given as

$$\bar{u} = \frac{1}{A_c} \iint u dA_c, \quad \tau_b = \frac{\iint u \tau dA_c}{\iint u dA_c}$$

In curvilinear coordinates (ξ, η) above expression becomes

$$\bar{u} = \frac{1}{A_c} \iint u \sqrt{g} d\xi d\eta, \quad \tau_b = \frac{\iint \sqrt{g} u \tau d\xi d\eta}{\iint \sqrt{g} u d\xi d\eta}$$

The product of local friction factor and Reynolds number ($f Re$) and Nusselt number Nu in dimensionless form are given as:

$$f Re = \frac{2D_h^2}{c\bar{u}}, \quad Nu = \frac{-D_h}{P_h \tau_b}$$

where D_h is the hydraulic diameter defined as

$$D_h = 4 \times \frac{\text{Area of Cross Section}}{\text{Wetted Perimeter}}$$

and P_h is the heated perimeter.

To validate the model, results were obtained for the limiting case of zero fin height so that the comparison can be carried out with the known results of the double pipe geometry. Table 1 shows a comparison of the computed results carried out by us with those of Kakac *et al.* [1987]. For the sake of comparison results are calculated for geometries of ratios of radii 0.05, 0.1, 0.25 and 0.5 in this limiting case only. The results obtained are in excellent agreement.

Table 1: Comparison of the limiting case results.

| \hat{R} | $f Re$ | | Nu | |
|-----------|----------|------------|----------|------------|
| | Computed | Literature | Computed | Literature |
| 0.05 | 21.5931 | 21.5675 | 17.7858 | 17.8113 |
| 0.10 | 22.3510 | 22.3429 | 11.8943 | 11.9058 |
| 0.25 | 23.3017 | 23.3018 | 07.7670 | 7.75347 |
| 0.50 | 23.8143 | 23.8125 | 06.1917 | 06.1810 |

The results of $f Re$ and Nu normalized by those of finless double pipe results are drawn in Figs. 4 and 5. The solid line shows the results of Syed [1997]. Again the results are comparable.

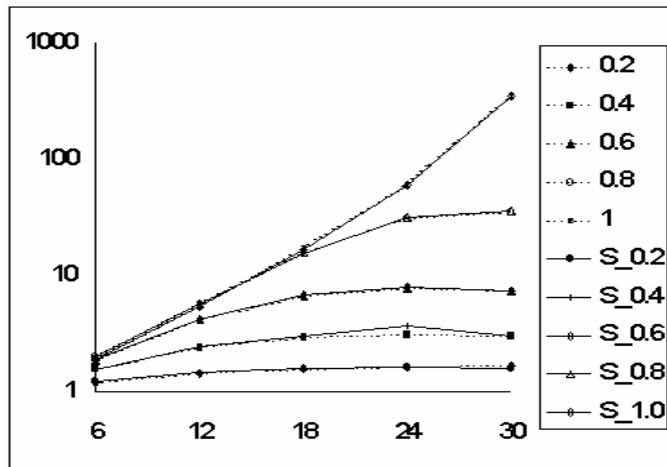


Fig. 4: Comparison of $f Re$ results.

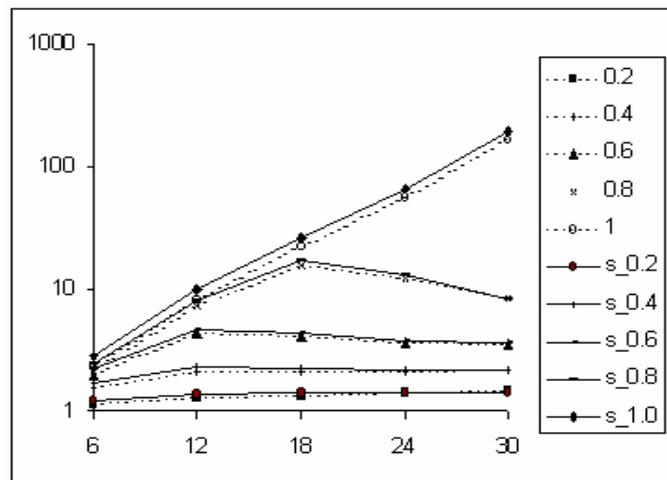


Fig. 5: Comparison of Nu .

CONCLUSION

The solution of coupled heat transfer equations generally requires a large amount of computational work both with the analytical and numerical methods. A study is carried out to reduce the amount of computational work and at the same time acquire the required order of accuracy. Body fitted grid is generated for this purpose and it is shown that the results obtained by solving the momentum and energy equations are in conformity with the existing results.

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