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RELIABILITY ANALYSIS OF FANS

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Abstract
Fans are an important component used in turbogenrator engine. The aim of this research is Reliability analysis of fans using Weibull and lognormal models for suspended samples. In this paper we analyze the current test design of fans. We develop reliability testing calculators that determine reliability values based on two different methods.

Keywords: Fans suspended life data, turbogenrator engine, Weibull and lognormal distributions.

INTRODUCTION

We develop reliability calculator useable for these products. This paper concentrates upon fans used in turbogenrator engine and explores two testing methods to evaluate their reliability. The first method uses the weibull distribution to model the time to failure. The second method uses the Lognormal distribution to model. We study these two methods of reliability testing. Following each discussion is an example of one data set showing the reliability computed at particular times. We also enumerate the pros and cons of each method which is best suitable for use. As part of this study, we develop two reliability calculators.

THEORETICAL BACKGROUND

TERMINOLOGY

We repeatedly use several terms throughout this paper that need to be clarified. The mean time to failure (MTTF) is the expected time that a fan will function before it fails for whatever reason. Conventionally, MTTF refers to non-repairable objects while mean time to between failures (MTBF) refers to repairable objects. For convenience, we are interested only in MTTF, The reliability of a fan at certain time T is the probability that the fan has never failed while continuously operating over a period of time T. Reliability is sometimes is called the survival probability. The term time for the purpose of this paper is in hours.

PROBABILITY DISTRIBUTIONS

There are two main functions considered when studying probability distributions, Probability density functions (pdf) and cumulative density functions (cdf). A pdf $f(x)$ has the following properties:

$$f(x) \geq 0 \text{ for all } x,$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1,$$

$$p(a < x < b) = \int_{a}^{b} f(x)\,dx,$$

Where $p(a < x < b)$ is the probability that x is between a and b. On the other hand, a cdf $F(x)$ is a function defined over the real number line with the properties:

$$F(x) = p(X \leq x) = \int_{-\infty}^{x} f(x)\,dx,$$

$x_1 < x_2$ implies $F(x_1) \leq F(x_2)$,

$$\lim_{x \to -\infty} F(x) = 0, \quad \lim_{x \to \infty} F(x) = 1$$

With these definitions in mind, we now discuss two distributions, first the weibull distribution, then the lognormal distribution. Since this study is concerned about the reliability of fans over time, then we shall write our random variable x in the equations above as x, where x is always greater than zero.

WEIBULL DISTRIBUTION

The Weibull probability distribution has two parameters η, β. It can be used to represent the failure probability density function (PDF) with time, so that:

$$f_w(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta - 1} e^{-\left(\frac{x}{\eta}\right)^{\beta}} \qquad (2.1)$$

$\eta > 0, \beta > 0, x > 0.$

Where β is the shape parameter (determining what the Weibull PDF looks like) and is positive and η is a scale parameter (representing the characteristic life at which 63.2% of the population can be expected to have failed) and is also positive.

The cumulative distribution function (CDF), denoted by $F_w(x)$, is:

$$F_w(x) = 1 - e^{-\left(\frac{x}{\eta}\right)^{\beta}} \qquad (2.2)$$

Note that $F_w(\eta) = 1 - (1/e)$ = 0.63212, which explains why we referred to η as 'characteristic life' or 'characteristic value' It represents the characteristic life of the fan.

$\beta < 1$:	high infant mortality , like a design or production flaw
$\beta = 1$:	random cause for failure(Weibull reduces to exponential distribution)
$1 < \beta < 4$:	engine operating environment(corrosive atmosphere)
$\beta \geq 4$:	failure mostly from age.

LOGNORMAL DISTRIBUTION

The probability density function (PDF) for 2-parameter lognormal distribution is:

$$f_L(x) = \frac{1}{\sqrt{2\pi} x\sigma} \exp\left\{ -\frac{[\ln x - \mu]^2}{2\sigma^2} \right\},$$

$$-\infty < \mu < \infty, \sigma > 0, x > 0. \qquad (2.3)$$

Where σ is the shape parameter, μ is the scale parameter. The units of σ and μ are the same as in the Weibull case. The restrictions on the values of σ and μ for the Lognormal distribution are as stated in equation (2.3). The corresponding Lognormal CDF is the integral of the PDF from 0 to time- to-failure *x*. It can be written in terms of the standard Normal CDF as:

$$F_L(x) = \Phi\left[\frac{\ln x - \mu}{\sigma}\right] \quad -\infty < \mu < \infty, \sigma > 0, x > 0. \qquad (2.4)$$

Where Φ (z) is the CDF of the standard Normal distribution defined as:

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{\frac{-u^2}{2}} du$$

Φ (z) is tabulated in many publications. The lognormal distribution is the normal distribution expect that the independent variable is $\ln x$ instead of x .

METHOD OF ANALYSIS

DESCRIPTION OF DATA

We select a particular data set to demonstrate two methods. Nelson [1982] presented data about engine fans. Here the x values in the data list represent actual engine fan life in hours. There are seventy samples in which fifty eight are suspended and twelve are failures. In the data listing suspended values are denoted by “S” while the failure values are abrivated as “F”.

WEIBULL ANALYSIS

The Weibull cumulative distribution function (CDF), denoted by *F(x)*, is:

$$F_w(x) = 1 - e^{-(\frac{x}{\eta})^{\beta}} \quad (3.1)$$

The liner form of the resulting Weibull CDF can be represented by a rearranged version of equation (3.1):

$$\ln x = \frac{1}{\beta} \ln\ln\left(\frac{1}{1-F_W(x)}\right) + \ln\eta$$

Comparing this equation with the liner form $y = BX + A$, leads to $y = \ln x$ and

$$X = \ln\ln\left\{ 1 \big/ 1 - F_W(x) \right\}.$$

If we minimize β and η by using the LS method then we obtain:

$$\hat{\beta} = \frac{n\sum_{i=1}^{n} X_i^2 - \left(\sum_{i=1}^{n} X_i\right)^2}{n\sum_{i=1}^{n} X_i y_i - \sum_{i=1}^{n} X_i \sum_{i=1}^{n} y_i}, \quad (3.2)$$

and

$$\hat{\eta} = \exp\left(\frac{\sum_{i=1}^{n} y_i}{n} - \frac{\sum_{i=1}^{n} X_i}{n\hat{\beta}}\right), \quad (3.3)$$

Where n is the sample size and ˆ indicates an estimate. The mathematical expression for X_i and y_i are:

$$X_i = \ln\ln\left[\frac{1}{1-F_W(x_i)}\right] \quad \text{and} \quad y_i = \ln x_i.$$

$F(x_i)$ can be estimated by using Benard’s formula, $\frac{i-0.3}{n+0.4}$, which is a good approximation to the median rank estimator [Abernethy 1994]. We uses the Benard’s median rank because it shows the best performance and it is the most widely used to estimate $F(x_i)$. The procedure for ranking suspended data is as follows:

1. List the time to failure data from small to large.
2. To rank the data with suspensions and use the equation below to determine the ranks, adjusted for the presence of the suspensions.

$$AdjustedRank = \frac{(ResverseRank) \times (PreviousAdjustedRank) + (n+1)}{(ReverseRank) + 1}$$

3. Use Benard's formula to assign median ranks to each failure.
4. Estimate the β and η by equations (3.2) and (3.3).

The $F(x_i)$ is estimated from the median ranks. Once $\hat{a}$ and $\hat{b}$ are obtained, then $\hat{\beta}$ and $\hat{\eta}$ can easily be obtained. After performing the regression then we compute

The mean life (also known as mean-time-to-failure MTTF) of the Weibull distribution is:

$$Mean_w = MTTF_w = \eta \Gamma\left(1 + \frac{1}{\beta}\right) \tag{3.4}$$

The variance of the Weibull distribution is:

$$VAR_w = \eta^2 \left[\Gamma\left(1 + \frac{2}{\beta}\right) - \left[\Gamma\left(1 + \frac{1}{\beta}\right) \right]^2 \right] \tag{3.5}$$

Where $\Gamma(x) = \int_0^\infty \frac{z^{x-1}}{e^z} dz$ is the Gamma function, which can be calculated with Lanczos' approximate formula [Lanczos 1964]. Note that the minimum value of the $\Gamma_{\min}(x)$=0.8856 when x$\cong$1.46. If $\beta = 1$, then the weibull cumulative distribution reduces to the cumulative exponential distribution with $\eta = MTTF$. To compute the reliability at time x, we compute $1 - F_w(x) = e^{-(\frac{x}{\eta})^\beta}$.

LOGNORMAL ANALYSIS

The Lognormal CDF is the integral of the PDF from 0 to time- to-failure *x*. It can be written in terms of the standard Normal CDF as:

$$F_L(x) = \Phi\left[\frac{\ln x - \mu}{\sigma}\right] \tag{3.6}$$

The Lognormal CDF, when plotted against appropriate probability axes, appears linear and so can be represented by a rearranged version of equation (3.6) as:

$$\Phi^{-1}(F(x)) = \frac{\ln x}{\sigma} - \frac{\mu}{\sigma} \tag{3.7}$$

Comparing this with the linear form y = BX+A, leads $y = \Phi^{-1}(F(x))$, X = $\ln x$ and $B = \sigma^{-1}$. The same LS procedure, as was used for the Weibull distribution. Where $y = \Phi^{-1}(F(x))$ the percentile of the standard Normal CDF is, $F(x_i) = \phi(x_p)$ can be estimated by using Bernard's formula. The same ranking procedure as was used for the Weibull distribution was also used for the lognormal distribution. The only difference is that step 3 should be replaced by the estimate of σ and μ .As before, the engine fan life data is taken to illustrate how to use this model. The rank table is the same as the Weibull rank table for the 2-parameter lognormal analyses for the engine fan life data. We compute the MTTF as

$$MTTF = e^{\mu + \frac{\sigma^2}{2}} \text{ and} \tag{3.8}$$

$$Var(MTTF) = e^{(2\mu + \sigma^2)}(e^{\sigma^2} - 1) \tag{3.9}$$

ANALYSIS OUTPUT AND RESULTS

WEIBULL AND LOGNORMAL ANALYSIS

For both Weibull and lognormal analysis, the table with one column a range of times in hours and the other column the corresponding reliabilities. For comparison, both methods will use the same data set. The Weibull method converting the recorded times into data points for linear regression using the above mention weibull analysis similarly the process using the lognormal method and data points determined by lognormal analysis. We perform linear regressions for the data points. From the linear regression of weibull analysis we estimate β and η . Similarly, we estimate σ and μ from the linear regression of lognormal analysis. The slope and the y-intercept for the regression line of Fig. 1 are 1.2788 and 1.6129E+4 respectively. We then obtain the weibull parameters $\beta = 1.2788$, $\eta = 1.6129E + 4$ and the correlation coefficient $\rho = 0.9854$. Using (3.4), the corresponding MTTF is 14939.16367 hours. From Fig. 2 we obtain the lognormal estimated values obtained were $\hat{\mu}$ = 10.0032, $\hat{\sigma}$ = 1.6376 and the correlation coefficient $\rho = 0.9878$ .Using (3.8), the corresponding MTTF is 84462.83381 hours. The weibull and lognormal probability plot is obtained using Weibull++7 as shown in the following Figures from 1-12. Here the horizontal scale is engine fan life data in hour's x . In Figs (1) and (2) the vertical scale is the cumulative density function (CDF), the proportion of the units that will fail up to age x in percent. The Statistical symbol for the CDF is $F(x)$, the probability of failure up to time x . In Figs (3) and (4) the vertical scale is the complement of cumulative density function $R(x) = 1 - F(x)$ is reliability, the probability of the units that will not failing up to time x in percent. Here in these graphs the doted points which show the failure components at the initial stage and the other components are survivors. The reliability function (RF), denoted by $R(x)$ (also known as the survivor function). In Figs (5) and (6) the vertical scale is the cumulative density function (CDF) is the unreliability, the proportion of the units that will fail up to age

X in percent. The Statistical symbol for the CDF is $F(x)$, the probability of failure up to time X. In Figs (7) and (8) the vertical scale is the failure rate (FR), The hazard function (HF) (also known as instantaneous failure rate), denoted by h(*t*), is defined as *f(t)/R(t)*. The units for *h*(*t*) are probability of failure per unit of time, in hours. Here β > 1, the hazard function is continually increasing which represents wear-out failures. In Figs (9) and (10) the horizontal scale is engine fan life data in hours X. The vertical scale is the failure and suspended time line. In this graph the star line shows the failure component while aero line shows the suspended components. In Figs (11) and (12) the horizontal scale is eta the characteristic life (η) and the vertical scale is beta (β) for the Contour Analysis for Engine fan life data. Here in these graph we find (99, 95, 90, 85, 75) $(1-\alpha)$ percent confidence interval is the range of values, bounded above and below, within which the true value is expected to fall. It measures the statistical precision of our estimate. The probability that the true value lies within the interval is either zero or one. Confidence is the frequency that similar intervals will contain the true values, assuming the fixed errors are negligible. If we use 90% or above confidence interval then it will contain the true value for reliability intervals. Here the likelihood contour plots for the parameter eta and beta do not intersect; this would indicate significant differences with confidence levels. But these confidence bounds are overlapping. The slope of the line β is particularly significant and may provide a clue to the physics of the survivors. We have age of the parts that are failed and suspended. Here age is the operating time. Here Beta >1 Implies wear-out period .The wear out shows the increasing hazard function. In an increasing hazard function new item in this life period has a smaller probability of failing than an old item. This shows that the fans may not initially have high failure rates. Therefore, during this period the failures are generally caused by the factors such as:

- Fatigue,
- Degradation,
- Wear,
- Electronic failures,

Here the failure modes for Fans are beta>1, and the Fans survives in wear out period, It shows from the span of life as the hazard rate increases with age.

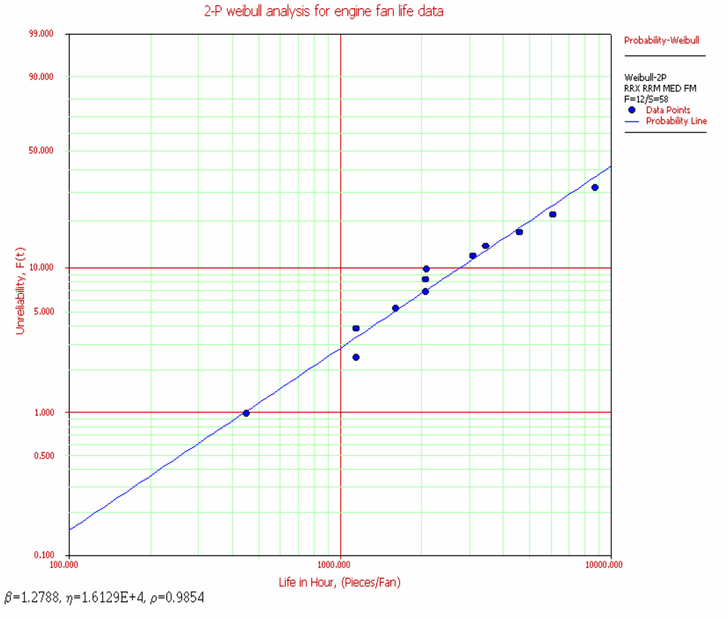

Fig. 1: 2-p Weibull Analysis for Engine fan life data

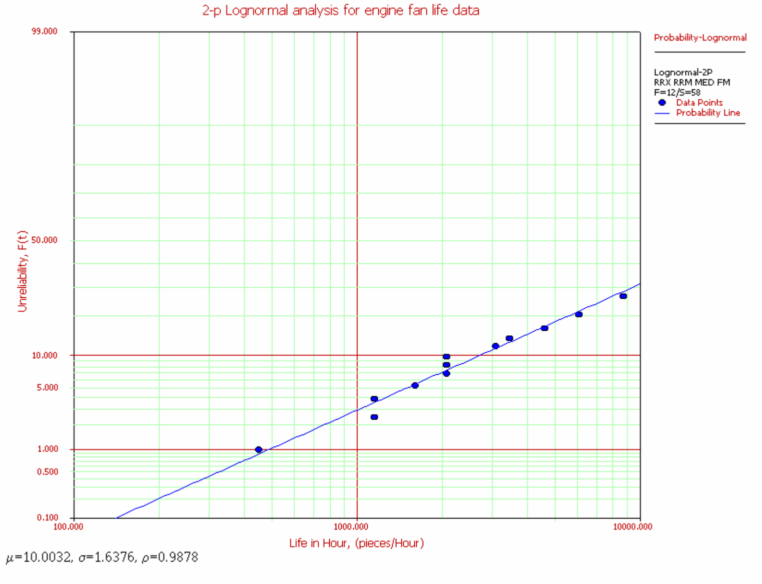

Fig. 2: 2-p Lognormal Analysis for Engine fan life data

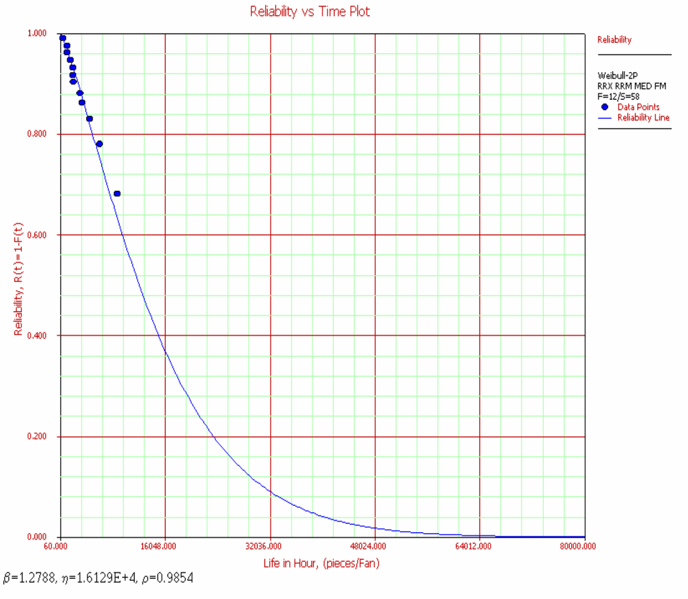

Fig. 3: 2-p Weibull Reliability Analysis for Engine fan life data

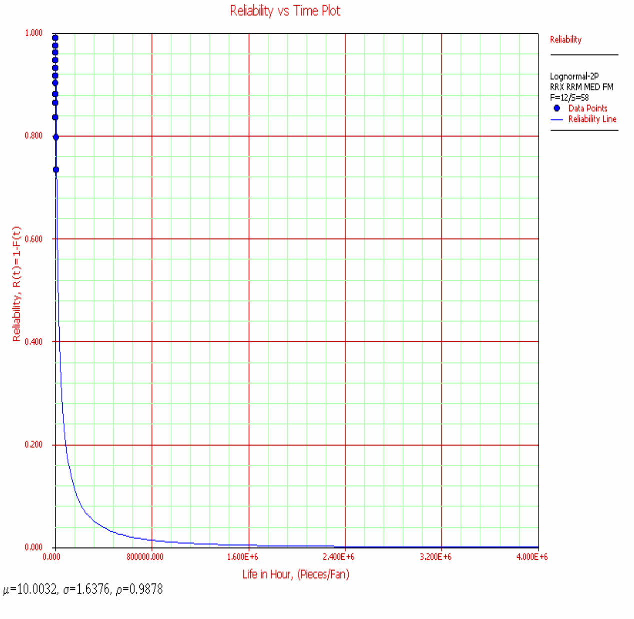

Fig. 4: 2-p Lognormal Reliability Analysis for Engine fan life data

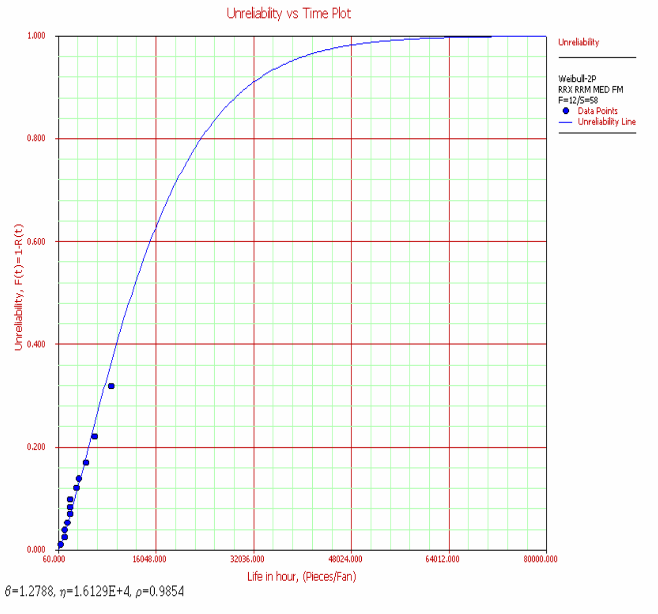

Fig. 5: 2-p Weibull Unreliability Analysis for Engine fan life data

Fig. 6: 2-p Lognormal Unreliability Analysis for Engine fan life data

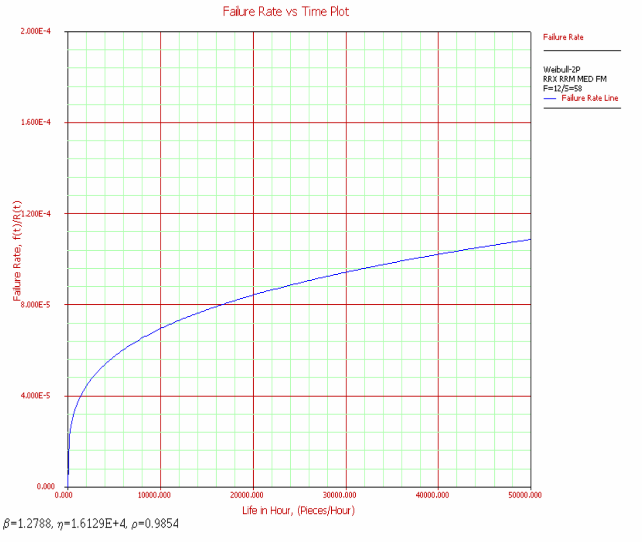

Fig. 7: 2-p Weibull Failure Rate Analysis for Engine fan life data

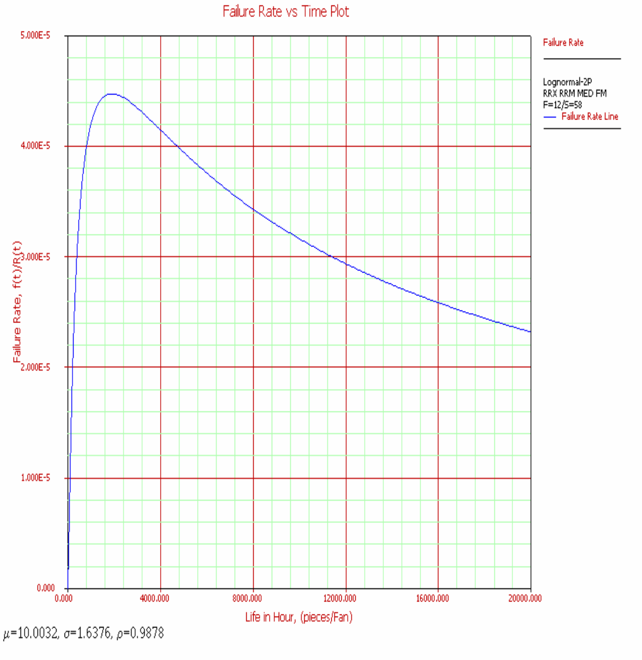

Fig. 8: 2-p Lognormal Failure Rate Analysis for Engine fan life data

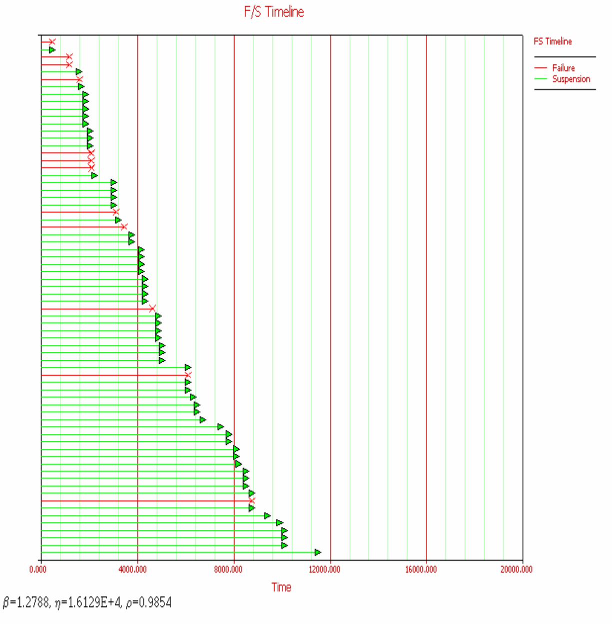

Fig. 9: 2-p Weibull Time Analysis for Engine fan life data

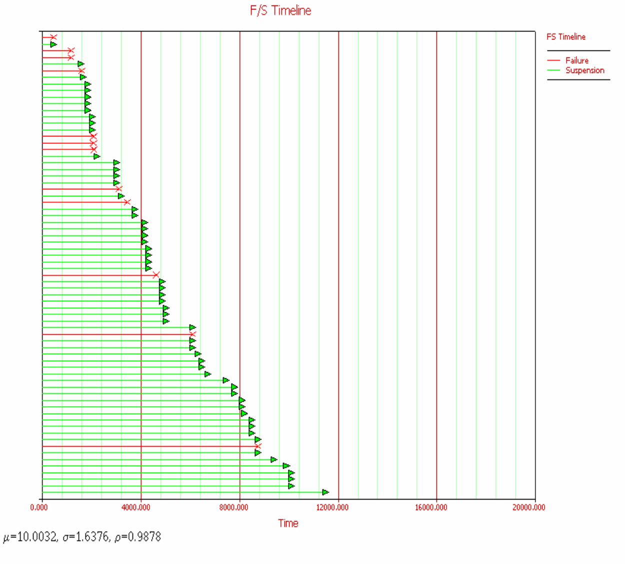

Fig. 10: 2-p Lognormal Time Analysis for Engine fan life data

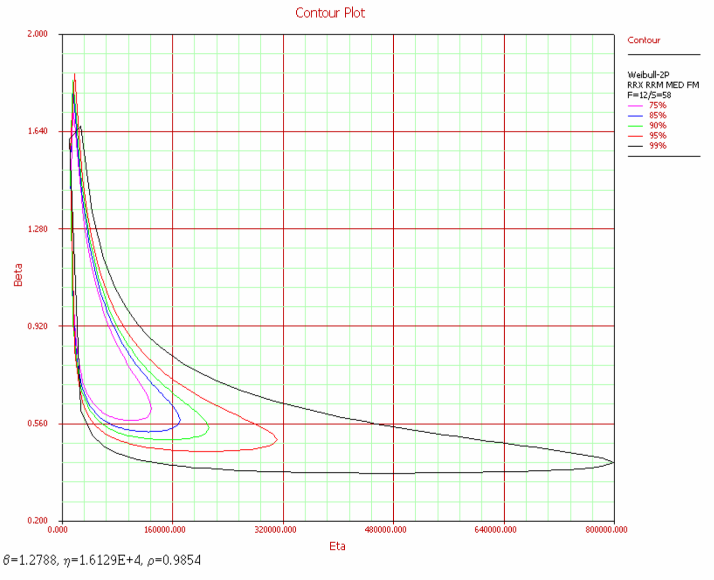

Fig. 11: 2-p Weibull Contour Analysis for Engine fan life data

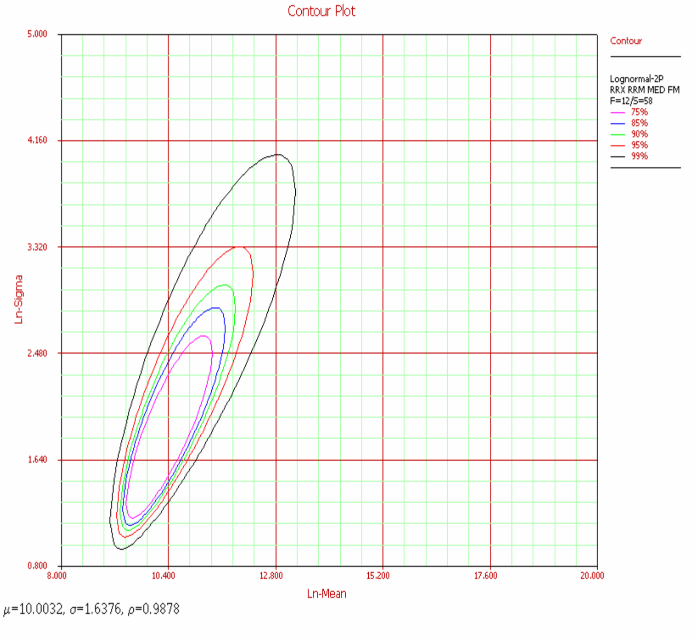

Fig. 12: 2-p Lognormal Contour Analysis for Engine fan life data

HOW TO COMPARE WEIBULL WITH LOGNORMAL

To compare Weibull to lognormal results, we consider which distribution more closely matches the data. One easy check is to compare the respective coefficients of determination R^2 for the regression lines. If the coefficient of determination is close to 1, then we have supporting evidence that the data points from linear relationship and hence the distribution is good model. For the Weibull line, $R^2 = 0.97101$, and for the lognormal line, $R^2 = 0.97575$ .The correlation coefficient for the Weibull distribution is $r = 0.985398$ and for the lognormal distribution $r = 0.987801$ Since the two correlation coefficients are closely matched, another possible tactic is to check the confidence intervals for reliability. The distribution with the shorter confidence intervals would ideally be the more accurate distribution for the given data set. Here we do not have the exact data points used in the regression because of median rank approximation. By using the likelihood contour plots for the parameter eta and beta we have clear bounds for the variances of all four parameters involved.

Table 1: Analysis for Engine Fan Life data in Hours

X	F/S	Frequency
450	F	1
460	S	1
1150	F	2
1560	S	1
1600	F	1
1660	S	1
1850	S	5
2030	S	3
2070	F	2
2080	F	1
2200	S	1
3000	S	4
3100	F	1
3200	S	1
3450	F	1
3750	S	2
4150	S	4
4300	S	4
4600	F	1
4850	S	4
5000	S	3
6100	F	1
6100	S	3
6300	S	1
6450	S	2
6700	S	1
7450	S	1
7800	S	2
8100	S	2
8200	S	1
8500	S	3
8750	F	1
8750	S	2
9400	S	1
9900	S	1
10100	S	3
11500	S	1

CONCLUSIONS

The Weibull distribution and the Lognormal distributions are extensively used in reliability and life testing. We have concluded that while comparing these two distributions, From the comparisons of the above results the Lognormal distribution provides slightly better results than the Weibull distribution. But we also note that from the correlation coefficients both the distributions are consistent for the Reliability analysis of fans.

References

Abernathy, R.B. (**1994**) "The New Weibull Handbook", SAE Professional Development.

Abernathy, R.B. (**2004**) "The New Weibull Handbook", Dept Stat. at Houston, Texas 77252-2608, USA.

Lanczos, C. (**1964**) "A Precision Approximation of Gamma Function", J. Siam Numer. Anal. **B 1**, 86-96.

Lawless, J.F. (**1982**) "Statistical Models and Methods of Life Time Data", *J Wiley*, New York, USA.

Nelson, W. (**1982**) "Applied Life Data Analysis\", *J. Wiley*, New York, USA.

Tobias, P.A. and Trindade, D.C. (**1986**) "Applied Reliability", Van Nostrand Reinhold Co. New York, USA, [ISBN: 0-442-28310-5].