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THE CONSTRUCTION OF SECOND ORDER NEIGHBOUR DESIGNS

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Abstract
This paper concerns the construction of second order neighbour designs. The idea of using the linear model in such designs will be introduced to construct the treatment balanced designs which are second-order neighbour balanced. The designs are constructed using variation of a simple method which is referred to as the method of cyclic shifts.

Keywords: Cyclic shift, neighbour block design, second order neighbour balanced design, treatment balanced design.

INTRODUCTION

Rees [1967] introduced the concept and name of 'neighbour designs'. The class of Rees neighbour designs includes schemes given in 1892 by Lucas [1957] for round dances. Rosa and Hawang [1975], Hwang and Lin [1974] and Bermand *et al* [1978] used the term balanced circuit designs for such designs. But none of the above such designs consider the second order neighbour. In this paper we will consider the construction of block designs for the case, where the plots in each block are arranged in a circle. Each plot, therefore, has two neighbours and a design is said to be first order neighbour-balanced if each treatment has every other treatment as a neighbour an equal number of times. Let this number be λ_1. A design is second-order neighbour-balanced if each treatment has every other treatment as one plot-away neighbour an equal number of times. Let this number be λ_2. The parameters of first-order and second-order neighbour designs are λ_1 and λ_2. These parameters are not independent and satisfy the following relation:

$$vr = bk, \quad \lambda_1 (v-1) = 2r, \quad \lambda_2 (v-1) = 2r \tag{1.1}$$

Here is an example of design which is second-order neighbour-balanced. For v=11 and k = 5

0	1	2	3	4	5	6	7	8	9	10
8	9	10	0	1	2	3	4	5	6	7
4	5	6	7	8	9	10	0	1	2	3
6	7	8	9	10	0	1	2	3	4	5
5	6	7	8	9	10	0	1	2	3	4

In the above design it is seen that in block 1 treatment 4 is the right second-order neighbour and treatment 6 is the left second-order neighbour of treatment 0. In block 8 treatment 1 is the right second-order neighbour and treatment 7 is the left second-order neighbour of treatment 0. Similarly it can easily be found the other second-order neighbour of treatment 0.

It will be explained how one can find the second-order neighbour count from the shifts in Theorem 3.1.

The mathematical model is

$$Y_{ij} = \mu + \tau_i + \beta_{(i+1)} + \beta_{(i-1)} + \gamma_{(i+2)} + \gamma_{(i-2)} + \alpha_j + \varepsilon_{ij}$$
$$i = 1, 2,..., k \quad j = 1, 2,...,b$$

where

μ is the general mean,

τ_i is the direct effect of the treatment applied to plot i,

$\beta_{(i+1)}$ is the first-order right-neighbour effect of the treatment on plot (i+1) mod (k+1)

$\beta_{(i-1)}$ is the first-order left-neighbour effect of the treatment on plot (i -1) mod (k+1)

$\gamma_{(i+2)}$ is the second-order right-neighbour effect of the treatment on plot (i+2) mod k

$\gamma_{(i-2)}$ is the second-order left-neighbour effect of the treatment on plot (i-2) mod k

α_j is the effect of block j

ε_{ij} is an independent random error with mean zero and variance σ^2.

In matrix terms, the model can be written as:

$$Y = X_0\mu + X_1\tau + X_2\beta + X_3\gamma + X_4\alpha + \varepsilon$$

where

Y is the bk x 1 vector of response.

X_0 is the bk x 1 vector of 1's.

X_1 is the bk x v incidence matrix for treatment effects.

X_2 is the bk x v incidence matrix for neighbour effects.

X_3 is the bk × v incidence matrix for second-order neighbour effects.

X_4 is the bk x b incidence matrix for block effects.

τ is the v x 1 vector of treatment effects.

β is the v x 1 vector of first-order neighbour effects.

γ is the v x 1 vector of second-order neighbour effects.

α is the b x 1 vector of block effects.

ε is the bk x 1 vector of random errors.

If one lets

$$X' = [\, X_0' : X_1' : X_2' : X_3' \,] \text{ and}$$
$$\Pi' = [\, \mu', \tau', \beta', \alpha' \,]$$

the model can be written as:

$$Y = X\hat{\Pi} + \varepsilon$$

The set of normal equations required to obtain the least squares estimate of $\hat{\Pi}$ is

$$(X'X)\,\hat{\Pi} = X'Y$$

Here $G = X_0'Y$, $T = X_1'Y$, $R_1 = X_2'Y$, $R_2 = X_3'Y$ and $B = X_4'Y$.

One can also note that

$X_0'X_0 = bk = rv$, $X_0'X_1 = rE_{1,v}$, , $X_0'X_2 = 2rE_{1,v}$, $X_0'X_3 = 2rE_{1,v}$, , $X_0'X_4 = kE_{1,b}$,
$X_1'X_1 = r\,I_v v$, $X_1'X_2 = L_1$, $X_1'X_3 = L_2$, $X_1'X_4 = N$, $X_2'X_2 = M_1$, $X_2'X_3 = M_2$,
$X_2'X_4 = 2N$, $X_3'X_3 = M_3$ $X_3'X_4 = 2N$, $X_4'X_4 = kI_b$

where $E_{1,v}$ is a matrix containing all 1's and is of dimension 1x v.

Under the restrictions

$E_{1,v}\,\tau = E_{1,v}\,\beta = 0$, $E_{1,v}\,\gamma = 0$, $E_{1,b}\,\alpha = 0$ and after simplification the adjusted LS estimates for τ, β, and γ are

$$\begin{pmatrix} rIv - (1/k)NN' & L_1 - (2/k)NN' & \mathbf{L_2} - (2/k)NN' \\ \mathbf{L_1} - (2/k)NN' & M_1 - (4/k)NN' & M_2 - (4/k)NN' \\ \mathbf{L_2} - (2/k)NN' & M_2 - (4/k)NN' & \mathbf{M_3} - (4/k)NN' \end{pmatrix} \begin{bmatrix} \hat{\tau} \\ \hat{\beta} \\ \hat{\gamma} \end{bmatrix} = \begin{pmatrix} T - (1/k)NB \\ R_1 - (2/k)NB \\ R_2 - (2/k)NB \end{pmatrix}$$

It will be noted that **NN'** is the treatment concurrence matrix whose (i, i')th element is the number of times treatments i and i' occur together in the same block, i ≠ i' and the (i, i')th element of $\mathbf{L}_1$ is the number of times treatment i' is a neighbour of treatment i, i ≠ i'. That one do not distinguish between right-and left-neighbours when constructing the matrix $\mathbf{L}_1$. $\mathbf{M}_1$ is a matrix whose (i, i') th element is the number of times treatment i and i' have a common neighbouring plot for i ≠ i'. For a first-order neighbour balanced design, all off-diagonal elements of $\mathbf{L}_1$ must be equal. For a treatment-balanced design all off-diagonal elements of **NN'** must be equal. The (i, i')th off-diagonal elements of L_2 gives the number of times each treatment pair (i, i') appears together as a second-order neighbours. A design is neighbour-balanced for second-order if the off-diagonal elements of L_2 are all equal. We can find the off-diagonal elements of L_2 by using theorem 3.1 (given in Methods of construction).
One also notes that M_3 is a matrix whose off-diagonal elements are the number of times each treatment pair has a second-order neighbouring plot in common and the (i, i')th element of M_2 is the number of plots which are the second-order neighbour of i'. It will be considered the construction of such designs from the view point of using cyclic shifts. The method is defined below. For more details see Iqbal [1991].

METHODS OF CYCLIC SHIFTS

The method of cyclic shift is a particular way of constructing a cyclic incomplete block design. This method is described here for a design with v treatments in b blocks of size k. The method of construction is described for one set of v blocks. Larger designs are constructed by combining the blocks of two or more full or partial sets of blocks. One can allocate to the first plot in ith block the treatment i, i=0, 1, 2, ..., v-1. This is denoted by using the vector $\mathbf{u_1}$=[0, 1, 2...], which holds the treatments allocated to the first plot in each of blocks 1,2,...,v, respectively. To obtain the treatment allocation of the remaining plots in each block, cyclically shift the treatment allocated to the first plot. In order to define a cyclic shift, let $\mathbf{u_i}$ denotes the 1×v vector which defines the allocation of the treatments to the ith plot in each block. That is, the jth element of $\mathbf{u_i}$ is the treatment allocated to plot i of block j. A cyclic shift of size q_i applied to plot i, is then such that $\mathbf{u_{i+1}}$ =[$\mathbf{u_i}$ + $\mathbf{q_i}$ **1**], where addition is mod v, **1** is a 1×v vector of ones, 1≤ i ≤ k-1 and 1≤ q_i ≤ v-1. Assuming that we always start with $\mathbf{u_1}$, as defined above, a design is completely defined by a set of k -1 shifts, **Q**, say, where **Q**=[q_1,q_2,...,q_{k-1}]. To avoid a treatment occurring more than once in a block one must ensure that, sum of any two successive shifts, the sum of any three successive shifts,..., sum of any k-1 successive shifts is not equal to zero mod v. Subject to this constraint, **Q** may consist of any combination of shifts including repeats.

To illustrate the above method of construction let us consider the construction of a design for v=6 and k=4. The set of shifts defined by **Q**=[q_1,q_2,q_3], where q_i=1,2,3,4,5, i=1,2,3. Suppose **Q** = [1, 2, 5]. Then $\mathbf{u_1}$=[0,1,2,3,4,5], $\mathbf{u_2}$=[1,2,3,4,5,0], $\mathbf{u_3}$=[3,4,5,0,1,2] and $\mathbf{u_4}$=[2,3,4,5,0,1]. The complete design is given below as Design 2.1.

Design 2.1

0	1	2	3	4	5
1	2	3	4	5	0
3	4	5	0	1	2
2	3	4	5	0	1

The properties of a design depend on the number of concurrences made between the pairs of treatments. Because of the cyclic nature of construction, the number of concurrences between any treatment and remainder can be obtained from the number of concurrences between treatment 0 and the remainder. Also the number of concurrences between treatment 0 and remainder can easily be obtained from **Q**. If shift q_1 and q_2, for example, are applied successively to treatment 0, the result is a concurrence between treatment 0 and treatment q_1 and q_2 and a concurrence between treatment 0 and treatment q_1+q_2. If a third shift, q_3, say, is then applied after q_1 and q_2, the following treatments will also concur with treatment 0: q_3, q_2+q_3 and $q_1+q_2+q_3$. This adding of shifts to get the treatments which concurs with 0 works for the general case and so enables the number of concurrences of a design to be obtained directly from the shift which defines it. In general case if shifts q_1, q_2,...,q_{i-1} have been applied successively to treatment 0, then the additional concurrences which results when shift q_i is applied are between treatment 0 and treatments q_i, q_i+q_{i-1},...,$q_2+q_3+...+q_i$, ,...,$q_1+q_2+...+q_i$. One also notes that the number of concurrences is "symmetric" about [v/2] in the sense that any shift of size q that results in a concurrence between treatment 0 and treatment q also results in a concurrence between treatment 0 and treatment (v-q) mod v.

In the design 2.1, q_1=1, q_2=2 and q_3=5 was used. From this one obtains q_1+q_2=3, q_2+q_3=7=1 (mod 6) and $q_1+q_2+q_3$=8=2 (mod 6). In the above design one appears twice (i.e. q_1=1 and q_2+q_3=1) and since 1 is symmetric to 5 and 5 appears once (i.e. q_3=5). Therefore, the concurrence between treatment 0 and treatment 1 is 3 and between treatment 0 and treatment 5 is also 3. Similarly 2 appears twice (i.e. q_2=2 and $q_1+q_2+q_3$=2), and since 2 is symmetric to 4, therefore, the concurrence between treatment 0 and treatment 2 is 2 and also between treatment 0 and treatment 4 is 2. And 3 appears once (i.e. q_1+q_2=3) and 3 is symmetric to itself, therefore, the concurrence between treatment 0 and treatment 3 is 2. Therefore, the concurrences between treatment 0 and treatments 1,2,3,4,5 are 3,2,2,2,3 respectively. The concurrences between treatment 1 and treatments 2,3,4,5 follow the same pattern i.e. the concurrences are 3,2,2,2. Similarly the concurrences between treatment 2 and treatments 3,4,5 are 3,2,2 and so on.

By using certain combinations of shifts we can construct designs that are made up of complete replicates of smaller designs. When v and k are not relatively prime, then partial sets of v/d blocks can also be obtained, where d is any common divisor of v and k. The shifts produce such partial sets of blocks can be obtained as follows. The smallest integer 'a' is found where (a×v)/k=n and n is an integer. Then if the sets of shifts used to construct the design is such that the sum of every 'a' successive shifts adds to n. The design will contain v/n blocks. It refers to designs which are constructed using such shifts as "fractional designs". These fractional designs are useful because one or more of their replicates can be added to another design or designs to obtain another design.

For v even and k even, one can also construct fractional designs by (i) setting the middle shift ($q_{k/2}$) equal to v/2 and ensuring that shifts q_i and q_{k-i} i=1,2,...(k/2 -1) are complement of each other. So, for example, if v=6 and k=4, a fractional design can be constructed by using the sets of shifts [2,3,4] or the set [1,3,5]. As an illustration of a fractional design, consider Design 2.2 below, which was constructed using the set [2,3,4]. It can be seen that the blocks can be grouped into two sets of three blocks: (0,2,5,3), (1,3,0,4) and (2,4,1,5). These three blocks constitute a half replicate of the design. Such fractional replicates are denoted by adding a fraction to the set of shifts used to construct the complete design. Therefore, it would denote the replicated set of three blocks in Design 2.2 by using the notation [2,3,4]1/2.

Design 2.2

0	1	2	3	4	5
2	3	4	5	0	1
5	0	1	2	3	4
3	4	5	0	1	2

In order to construct a design with more than v blocks, one can combine the blocks obtained from more than one set of shifts. As an example, Design 2.3 below is a design for 6 treatments in 15 blocks of 4 which has been constructed by combining together the blocks which are obtained from the three sets of shifts [1,1,2], [1,1,3] and [2,3,4]1/2. In short notation one would say Design 2.3 had been constructed by using shifts [1,1,2]+[1,1,3]+[2,3,4]1/2. Where the "+" sign indicates that the blocks constructed from the separate sets of shifts must be combined together.

Design 2.3

0	1	2	3	4	5	0	1	2	3	4	5	0	1	2
1	2	3	4	5	0	1	2	3	4	5	0	2	3	4
2	3	4	5	0	1	2	3	4	5	0	1	5	0	1
4	5	0	1	2	3	5	0	1	2	3	4	3	4	5

In next section it will be explained how to choose the sets of shifts which give neighbour designs in circular blocks.

METHODS OF CONSTRUCTION

If treatment 0 is on plot i in block j, then the treatment a, say, on plot (i+1) mod (k+1) is the right-neighbour of treatment 0 in that block. The treatment on plot (i-1) mod (k+1) in block j, is the left-neighbour of treatment 0 in block j. The cyclic method of construction ensures that if any treatment, a say, is a right-neighbour of treatment 0 in block j, then treatment v-a mod(v) will be left-neighbour of treatment 0 in some other block j', j' ≠ j. This property makes the counting up of the right- and left-neighbours of treatment 0 quite easy. If a shift a, say, is applied to treatment 0, then treatment ‘a’ will be a right- neighbour of treatment 0. In some other block treatment v-a mod (v) will be a left-neighbour of treatment 0. To give it a name, one calls v-a mod (v) the **complement** of shift q. Therefore, given a set of shifts **Q**, one can determine all the right- and left-neighbours of treatment 0 from the shifts contained in **Q**. The shifts q_1, q_2,...,q_{k-1} define the right-neighbours of treatment 0 and q_1+q_2+...+q_{k-1}, mod (v) defines the left-neighbour of treatment 0 in the first

block. By taking the complements of these k shifts, one gets all the remaining neighbours of treatment 0.
The conditions that the shifts must satisfy to produce a neighbour-balanced design will be given in Theorem 3.1 and to produce also a second-order neighbour balanced design is given in Theorem 3.1 below:

THEOREM 3.1

A design is second-order neighbour-balanced if each shift appears an equal number (λ_2) of times in a new set of shifts where this new set of shifts consist of (i) the sum of every two and (k-2) successive shifts and (ii) the complements of the shifts in (i).

Proof

Treatment 0 in row i is the left-second-order-neighbour of the treatment in row (i+2) mod k if $i \le k - 2$ and can be obtained by adding the corresponding two successive shifts (i.e. the shifts used to construct the (i+1)th and (i+2)th row). For i = k-1, the left-second-order neighbour can be found as complement of the sum of k-2 successive shifts $q_1, q_2, \ldots\ldots q_{k-2}$ and for i = k, it can be found as the complement of the sum of the k-2 successive shifts $q_2, q_3, \ldots\ldots q_{k-1}$. Similarly treatment 0 in row i, i>2 is the right second-order neighbour of the treatment in row i-2 and can be obtained by taking the complement of the sum of every two successive shifts. If $i \le 2$ treatment 0 is the second-order right-neighbour to the treatment in row (k-2+i) and can be obtained by taking the sum of every k-2 successive shifts. Since these two contain all the values of list(i) and (ii), the design will be second-order neighbour-balanced with $\lambda_2 = 2r/(v-1)$.
One can also find the off-diagonal elements of M_1, M_2 and M_3 from the set of shifts. To obtain the off-diagonal elements of the matrix M_1, one constructs the lists (i) and (ii) which are defined as follows. To obtain the elements in list (i) add together each successive pair of shifts and enter their sum in the list. In list (ii) enter the sum of each successive set of (k-2) shifts, take the complement of each number of list (i) and (ii), then count the number of times each of the treatment labels 1,2,…….,v-1 occurs in the list (i) and (ii) and their complements. These counts are the off-diagonal elements in the first row of M_1. The remaining rows can then be obtained by cyclically shifting the entries in the first row. One can also find the off-diagonal elements of M_2 and M_3 from the shifts, however, there is no straight forward rule which is true for all values of k. Therefore, consider some different values of k and give the new set of shifts, which give the off-diagonal elements of M_2 and M_3.
First consider M_2. For each of the following values of k one finds the following two sets of new shifts and then counts the number of times each treatment label appears in the lists. These will be the off-diagonal elements of M_2.

a) k = 4

New set (i) $q_1, q_2, q_3,\ q_1+q_2+q_3$.

New set (ii) is the complement of elements in set (i).

b) k = 5

New set (i) $q_1, q_2, q_3, q_4,\ q_1+q_2,\ q_2+q_3,\ q_3+q_4,\ q_1+q_2+q_3,\ q_2+q_3+q_{4,},\ q_1+q_2+q_3+q_4$.

New set (ii) is the complements of elements in set (i).

c) k = 6

New set (i) $q_1, q_2, q_3, q_4, q_5,\ 2(q_1+q_2+q_3),\ 2(q_2+q_3+q_4),\ 2(q_3+q_4+q_5),\ q_1+q_2+q_3,\ q_5,\ q_6$.

New set (ii) is the complement of elements in set (i).

d) k = 7

New set (i) q_1,q_2,q_3,q_4,q_5,q_6, q_1+q_2, q_3, q_2+q_3, q_4, q_3+q_4, q_5, $q_4+q_5+q_6$, $q_1+q_2+q_3$, q_4, $q_2+q_3+q_4+q_5$, $q_3+q_4+q_5+q_6$, $q_1+q_2+q_3+q_4+q_5+q_6$

New set (ii) is the complement of elements in set (i).

Similarly, now consider M_3 and have the following new sets of shifts (However, for k = 4, one can have that M_3 = 0 and so no sets of shifts are given).

a) k = 5

New set (i) q_1,q_2,q_3,q_4, $q_1+q_2+q_3+q_4$

New set (ii) is the complement of elements in set (i).

b) k = 6

New set (i) q_1+q_2, q_2+q_3, q_3+q_4, q_4+q_5, $q_1+q_2+q_3+q_4$, $q_2+q_3+q_4+q_5$,

New set (ii) is the complement of elements in set (i).

c) k = 7

New set (i) $q_1+q_2+q_3$, $q_2+q_3+q_4$, $q_3+q_4+q_5$, $q_4+q_5+q_6$, $q_1+q_2+q_3+q_4$, $q_2+q_3+q_4+q_5$, $q_3+q_4+q_5+q_6$.

New set (ii) is the complement of elements in set (i).

ADDING AN ADDITIONAL TREATMENT

Sometimes a design for v - 1 treatments with block sizes of k and k-1 can be converted into a neighbour-balanced design for v treatments, and all blocks of size k, by adding an additional treatment to each of the smaller blocks of size k-1. Design 3.1.1 below, is an example of such a design for v = 6, k =3 and λ_1 = 2, λ_2 =2 with set of shifts $\mathbf{Q_1}$ = [1,1] and $\mathbf{Q_2}$ = [1]t.

Design 3.1.1

0	1	2	3	4	0	1	2	3	4
1	2	3	4	0	2	3	4	0	1
2	3	4	0	1	5	5	5	5	5

The following Table 3.1 consists of some designs which are first-order as well as second-order neighbour balanced. These designs are also treatment balanced except the designs for v = 12, 13 and k = 6.

Table 3.1: First-order as well as second-order neighbour balanced designs.

v	**k**	**Sets of Shifts**	**λ_1**	**λ_2**	**Con**
4	3	[1,2]	2	2	2
5	3	[1,1]+[2,2]	3	3	3
6	3	[1,1]+[2]t	2	2	2
7	3	[1,2]	1	1	1
8	3	[1,1]+[1,2]+[1,4]+ [2,2]+[2,5]+[3,3]+ [3,4]	6	6	6
9	3	[1,2]+[4]t(½)	1	1	1
10	3	[1,3]+[2,2]+[3,3]⅓ [1]t	2	2	2
11	3	[1,1]+[2,2]+[3,3]+ [4,4]+[5,5]	3	3	3

v	k	Set of shifts	λ_1	λ_2	Con
12	3	[1,2]+[1,5]+[2,3]+ [4,4]⅔	2	2	2
13	3	[1,3]+[2,5]	1	1	1
15	3	[1,3]+[2,6]+[5,5]⅓	1	1	1
16	3	[1,3]+[1,6]+[2,3]+ [2,6]+[5,7]	2	2	2
17	3	[1,1]+[2,2]+….. …...+[8,8]	3	3	3
19	3	[1,5]+[2,8]+[3,4]	1	1	1
21	3	[1,9]+[2,4]+[3,5]+ [7,7]⅓	1	1	1
23	3	[1,1]+[2,2]+…… …...+[11,11]	3	3	3
25	3	[1,5]+[2,8]+[3,9]+ [4,7]	1	1	1
27	3	[1,4]+[2,10]+[3,8]+ [6,7]+[9,9]⅓	1	1	1
29	3	[1,1]+[2,2]+……. …..+[14,14]	3	3	3
31	3	[1,14]+2,10]+[3,8]+ [4,5]+[6,7]	1	1	1
37	3	[1,13]+[2,3]+[4,11]+ [6,10]+[7,12]+[8,9]	1	1	1
4	4	[2,1,2]½+[1,1,1]1/4	2	2	3
5	4	[1,3,4]	2	2	3
6	4	[1,2,5]+[1,3,4]+ [1,3,5]½	4	4	6
7	4	[1,1,1]+[2,2,2]+ [3,3,3]	4	4	6
8	4	[1,2,3]+[2,4]t	2	2	3
9	4	[1,2,3]+[2,5,3]+ [2,4,4]+[4,1,6]	4	4	6
10	4	[1,2,4]+[1,4,2]+ [1,6,5]+[1,7,4]+ [3,5,7]½	4	4	6
11	4	[1,1,1]+[2,2,2]+…. …….+ [5,5,5]	4	4	6
12	4	[2,7,5]+[5,2,3]+ [1,7]t	2	2	3
13	4	[1,2,6]+[3,4,8]+ [8,1,10]	2	2	3
14	4	[1,3,2]+[1,5,11]2+ [3,9,6]2+[1,3]t+ [2,6]t	4	4	6
16	4	[1,6,5]+[3,7,7]+ [4,2,10]+[6,10] t	2	2	3
5	5	[1,1,1,1]1/5 + [2,2,2,2]1/5	1	1	2

6	5	[2,2,2,2]1/5 +			
		[1,2,1] t	2	2	4
7	5	[1,1,1,1]+[2,2,2,2]+			
		[3,3,3,3]	5	5	10
9	5	[1,1,3,2]+[1,2,3,5]+			
		[3,3,4,4]+[3,4,1,2]	5	5	10
10	5	[1,2,3,5]+[4,1,6] t	2	2	4
11	5	[8,7,2,10]	1	1	2
7	6	[1,1,2,2,4]	2	2	5
9	6	[1,5,4,3,7]½ +			
		[1,2,2,5] t	2	2	5
12	6	[1,2,3,4,5]+			
		[5,10,2,3] t	2	2	4,5,6
13	6	[6,3,5,9,2]	1	1	2,3
7	7	[1,1,1,1,1,1]1/7+….			
		….+[3,3,3,3,3,3]1/7	1	1	3
8	7	[2,6,5,4,1] t +			
		[4,4,4,4,4,4]1/7	2	2	6

All the designs given in the above Table 3.1 are new. No one has considered such designs before. In all cases one can find the properties of a design from the sets of shifts used to construct the designs instead of constructing the actual designs.

References

Bermand, J.C., Hwang, C, and Sotteau, D. (**1978**) “Balanced cycle and circuit designs: even cases”, *Discrete Mathematics*, **12**, 269-293.

Hwang, F.K. and Lin, S. (**1974**) “A direct method to construct triple system”, *J. Combin. Theory*, **A 17**, 84-94.

Iqbal, I. (**1991**) “Construction of experimental design using cyclic shifts”, Ph.D. Thesis, University of Kent at Canterbury, U.K.

Locus, H.L. (**1957**) “Extra-period Latin square change-over designs”, *J. Dairy Sci.*, **40**, 225-239.

Rees, H.D. (**1967**) “Some designs of use in Serology”, *Biometrics*, **23,** 119-155.

Rosa, A. and Hawang, C. (**1975**) “Another class of balanced graph design: balanced circuit designs”, *ARS Combinatoria*, **5**, 293-318.