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FORECASTS OF A SET OF FUTURE OBSERVATIONS OF PAKISTAN'S PRIVATE CONSUMPTIONS AND INVESTMENTS: A BAYESIAN APPROACH UNDER M-STAGE MODEL

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Abstract: In management and administration, the need for planning and control is important because lead time for managerial decision making ranges from several years or more to a few days or even hours. Information about future events from forecast is usually a critical input into a wide range of managerial and administrative decision-making process. In this paper the problem of forecasting the private consumptions and investments of Pakistan where the prior knowledge may be derived in M-stage in the manner first introduced by Lindley and Smith [1972] have been discussed. The predictive distribution of the future set of observations is derived under the M-stage model and forecast of the private consumptions and investments of Pakistan for 1981-82 to 1987-88 have been computed.

Keywords: Bayesian Inference, Forecasting, M-Stage Model, Informative Prior, Posterior and Predictive Distributions, Predictive Interval, Variance-Covariance Matrix.

INTRODUCTION

Two key functions of management for any enterprise or institution are planning and control. Firms plan for the future and planning for future involves a series of steps. One such series of steps is as follows:

1. Determine the product and geographic markets where the firm can achieve the goals of profit maximization and perhaps, the largest contribution to society.
2. Forecast demand in these markets under different conditions of promotion, price competition and general economic activity.
3. Forecast the costs of production and marketing at different level of output under the conditions of changing technology and price.
4. Choose the best plan of operations, which maximizes the well being of the firm.
5. Implement personnel training programs, capital constructions and acquisition programs to carry out the general corporate plan.

Information about future events from forecast, are usually a critical input into a wide range of managerial and administrative decision making, since today's plans are dependent on future expectation. Forecasts are numerical estimates of future level of sales, demands, inventories, costs, imports, exports, and prices, among others for a firm, an industry, a sector of economy or the total economy. The objective of forecasting is to assist management to plan requirements for marketing efforts, materials, personnel, production, services, capital acquisition and construction, and finances [Blazey 1976, Jenkins 1992]. Forecasting is important to the firm

i) The firm faces fluctuation in demand and inventory level throughout the year.
ii) Firm's sales are related to change in the economic environment in which it operates and
iii) Abrupt changes in the economic conditions have serious consequences for the firm. Implementation of a forecasting program can enable the firm to plan and react promptly in the future and keep it on a profitable.

Forecasting methods to produce numerical estimate range from relatively simple technique to complex and sophisticated techniques e.g. Judgmental methods [Sanders and Ritzman 1995], Moving average, exponential smoothing [Battery 1969], decomposition methods, Regression Analysis, Econometric Modeling, Adaptive filtering, Box-Jenkin's technique, Economic Indicators, Delphi Method, forecast revision [Groff 1973, Klein and Moore 1982, McNees 1982, Young 1982, Makridakis *et al.* 1983, Hanke and Reitsch 1986, Jaffrey 1987].

This paper discusses the problem of prediction based on experimental data taken according to certain well-known experimental plans, and where the prior knowledge may be derived in M-stage in the manner first discussed by Lindley and Smith [1972]. The selection of the values of parameters [Guttman 1969, Hoeral *et al.* 1975] by Lindley and Smith's M-stage model has attracted for the present study to forecast the future set of observations based on this model.

Guttman [1969] in discussing the role of prediction has stated that the model of the future observation does not change, because he (the experimenter) will be involved with the process again and that he wishes to have same idea of the behavior of future realization from the process. We believe that in many experimental situations that the experimenter is indeed in the situation where he/she will be addressing the process again and prediction of the response where the controlled variables are set to levels used in the previous experiment or to be set at new treatment combinations is of prominent importance.

This paper, then answers that we are in real situations, and proceeds to calculate the relevant predictive distribution and its properties. In general, as is well known, the predictive distribution h of $\underline{y}_f$, given experimental data $\underline{y}_e$ (where the subscript f stands for future and subscript e stands for experimental) is

$$h(\underline{y}_f / \underline{y}_e) = \int_{\underline{\theta}} f(\underline{y}_f / \underline{\theta}).P(\underline{\theta} / \underline{y}_e)\, d\underline{\theta} \qquad (1.1)$$

Where $P(\underline{\theta} / \underline{y}_e)$ is the posterior distribution of $\underline{\theta}$, given the experimental data $\underline{y}_e$ collected using an experiment e. In this paper we will compute the needed posterior using experiment e and prior information. Using

(1.1) expectation of $\underline{y}_f$ will then be calculated and used as predictors. The accuracy as measured by their Variance-Covariance, will then be assessed. In section 3, we apply these derived results to the private consumptions and investments data of Pakistan.

PREDICTIVE DISTRIBUTION VARIANCE-COVARIANCE MATRICES KNOWN

We start off by quoting a key result of the paper by Lindley and Smith [1972, p. 6]

THEOREM 2.1: (Lindley and Smith)

Suppose that, given $\underline{\theta}_1$, the (Nx1) random vector $\underline{y}_e$ is such that

$$\underline{y}_e \sim N(A_1 \underline{\theta}_1, C_1) \tag{2.1}$$

and given $\underline{\theta}_2$,

$$\underline{\theta}_1 \sim N(A_2 \underline{\theta}_2, C_2) \tag{2.2}$$

and given $\underline{\theta}_3$

$$\underline{\theta}_2 \sim N(A_3 \underline{\theta}_3, C_3) \tag{2.3}$$

where C_i are all non-singular.

Then, the posterior distribution of $\underline{\theta}_1$, given $\{A_i\}$,$\{C_i\}$, $\underline{\theta}_3$ and $\underline{y}_e$ is

$$P(\underline{\theta}_1 / \{A_i\}, \{C_i\}, \underline{\theta}_3, \underline{y}_e) \sim N(D\underline{d}, D) \tag{2.4}$$

with $$D^{-1} = A_1' \, C_1^{-1} A_1 + [C_2 + A_2 C_3 \, A_2']^{-1} \tag{2.5}$$

and $$\underline{d} = A_1' \, C_1^{-1} \, \underline{y}_e + [C_2 + A_2 C_3 \, A_2']^{-1} \, A_2' \, A_3 \underline{\theta}_3 \tag{2.6}$$

The proof and much interesting discussion are given in Lindley and Smith [1972]. Now suppose that it is planned to take a future set of observations $\underline{y}_f$, where $\underline{y}_f$ is to be generated as $\underline{y}_e$ generated in (2.1), that is, by $N(A_1^* \, \underline{\theta}_1, C_1^*)$ process. (C_1^* is of the same form as C_1, but is possibly of different order, determined by the order of $\underline{y}_f$ and /or the number of rows of A_1). We now derive the predictive distribution of $\underline{y}_f$, given $\underline{y}_e$, using (1.1) with $h(\underline{y}_f /(A_1^*, \underline{\theta}_1, C_1^*)$ as implied above and $P(\underline{\theta}_1 / \underline{y}_e, \{A_i\}, \{C_i\}, \underline{\theta}_3\}$ as in (2.4), we find that the predictive distribution is such that;

$$h(\underline{y}_f / \underline{y}_e) = k.\int_{\underline{\theta}_1} \exp\{-1/2(\underline{y}_f - A_1^* \, \underline{\theta}_1)' \, C_1^{*-1} (\underline{y}_f - A_1^* \, \underline{\theta}_1)\} \exp\{-1/2(\underline{\theta}_1 - D\underline{d})' \, D^{-1}(\underline{\theta}_1 - D\underline{d}) \, d\underline{\theta}_1 \tag{2.7}$$

where D and $\underline{d}$ are defined in theorem (2.1).

To prepare the integrand of (2.7) for integration with respect to $\underline{\theta}_1$, we first write quadratic from, apart from the term –1/2, which appears in the exponent of the integrand as follows:

$[\underline{y}_f - A_1^* \underline{\theta}_1]' C_1^{*-1} [\underline{y}_f - A_1^* \underline{\theta}_1) + [\underline{\theta}_1 - D\underline{d}]' D^{-1}[\underline{\theta}_1 - D\underline{d}]$

$= \underline{\theta} \ [(A_1^{*\prime} C_1^{*-1} A_1^*] \underline{\theta}_1 - 2\underline{\theta}_1' (A_1^{*\prime} C_1^{*-1} \underline{y}_f + \underline{y}_f' C_1^{*-1} \underline{y}_f + \underline{\theta}_1' D^{-1} \underline{\theta}_1 - 2\underline{\theta}_1' \underline{d}$

$+\underline{d}' D\underline{d}$

$= \underline{y}_f' C_1^{*-1} \underline{y}_f - (A_1^{*\prime} C_1^{*-1} \underline{y}_f + \underline{d})' U(A_1^{*\prime} C_1^{*-1} \underline{y}_f + \underline{d})$

$= \underline{y}_f' [C_1^{*-1} - C_1^{*-1} A_1^* U A_1^{*\prime} C_1^{*-1}] \underline{y}_f - 2\underline{y}_f' C_1^{*-1} A_1^* U\underline{d} - \underline{d}' U d$

$= \underline{y}_f' U^{*-1} \underline{y}_f - 2\underline{y}_f' \underline{u}^* - \underline{d}' U\underline{d}$ (2.8)

where we have set

$U^{*-1} = C_1^{*-1} - C_1^{*-1} A_1^* U A_1^{*\prime} C_1^{*-1}, \quad \underline{u}^* = C_1^{*-1} A_1^* U\underline{d}$ (2.9)

Hence (2.8) may be written as

$(\underline{y}_f - U^* \underline{u}^*)' U^{*-1} (\underline{y}_f - U^* \underline{u}^*) - [\underline{u}^{*\prime} U^* \underline{u}^* + \underline{d}' U\underline{d}]$ (2.10)

We note that the quantity in square bracelets [ ] of (2.10) is constant and therefore, from (2.10) we have, given $\underline{y}_e$

$\underline{y}_f \sim N(U^* \underline{u}^*, U^*)$ (2.11)

Where U^* and $\underline{u}^*$ are given in (2.9).

$U^{*-1} = C_1^{*-1} - C_1^{*-1} A_1^* U A_1^{*\prime} C_1^{*-1}$

$= C_1^{*-1} - C_1^{*-1} A_1^* (A_1^{*\prime} C_1^{*-1} A_1^* + D^{-1}) A_1^{*\prime} C_1^{*-1}$ (2.12)

and an easy application of the matrix lemma of Lindley and Smith [1972 (p. 5, expression 10)] yields

$U^* = C_1^{*-1} + A_1^* D A_1^{*\prime} = D^*$ (2.13)

Further, from (2.11), we have that

$E(\underline{y}_f / \underline{y}_e) = U^* \underline{u}^* = U^* (C_1^{*-1} A_1^{*\prime} U) \underline{d}$

$= D^* C_1^{*-1} A_1^* [A_1^{*\prime} C_1^{*-1} A_1^* + D^{-1}]^{-1} \underline{d}$

$= D^* \underline{d}^*$

where $\underline{d}^* = C_1^{*-1} A_1 [A_1^{*\prime} C_1^{*-1} A_1^* + D^{-1}]^{-1} \underline{d}$ (2.14)

Hence, we have

THEOREM 2.2

Suppose $\underline{y}_e$ is generated as in theorem (1.1) and $\underline{y}_f$ is generated from the process governing (2.1), but with different design matrix A_1^* (where

A_1^* is not necessarily equal to A_1) so that $\underline{y}_f / \underline{\theta}_1 \sim N(A_1^* \underline{\theta}_1, C_1^*)$, then the predictive distribution h of $\underline{y}_f$, given $\underline{y}_e$ and $\{A_i\}$, $\{C_i\}$, $\underline{\theta}_3$, A_1^*, C_1^* is

$$\underline{y}_f \sim N(D^* \underline{d}^*, D^*)$$

Where D^*, $\underline{d}^*$ are given in (2.13) and (2.14) respectively. In the subsequent subsection, we specialize for the case where $\underline{y}_f \sim N(A_1\underline{\theta}_1, C_1)$, that is, the same design matrix A_1 is to be used to generate $\underline{y}_f$ as was used to generate $\underline{y}_e$ and subject to the same experimental error, described by $V(\underline{y}_e / \underline{\theta}_1) = C_1$. Certain simplification and interesting results then come out, in regard to $E(\underline{y}_f / \underline{y}_e)$ and the variance covariance matrix $V(\underline{y}_f / \underline{y}_e)$ of the predictive distribution $\underline{y}_f$, given, in summary, that

$$E(\underline{y}_f / \underline{y}_e) = D^* \underline{d}^*, \quad V(\underline{y}_f / \underline{y}_e) = D^* \qquad (2.15)$$

Of general interest is the case where the prior distribution of $\underline{\theta}_2$ is non-informative, that is, the precision matrix $C_3^{-1} = 0$, we have the following corollary to theorem (2.2)

COROLLARY 2.2.1

Under the assumptions given in theorem (2.1) and (2.2), and if $C_3^{-1} = 0$, where 0 denotes a matrix of zeroes, then the predictive distribution of $\underline{y}_f$ given $\underline{y}_e$, is

$$N(D^o \underline{d}^o, D^o)$$

Where

$$D^o = C_1^{-1} + A_1 [A_1^{*/} C_1^{-1} A_1 + C_2^{*-1} - C_2^{*-1} A_2 (A_2' C_2^{*-1} A_2)^{-1} . A_2' C_2^{*-1}]^{-1} A_1^* \qquad (2.16)$$

And

$$\underline{d}^o = C_1^{-1} A_1 [A_1^{*/} C_1^{-1} A_1 + C_2^{*-1} - C_2^{*-1} A_2 . (A_2^* C_2^{*-1} A_2)^{-1} . A_2' C_2^{*-1})]^{-1} A_1^* C_1^{-1} \underline{y}_e \qquad (2.17)$$

Further the expectation of the predictive distribution may be written as;

$$E(\underline{y}_f / \underline{y}_e) = D^o \underline{d}^o$$
$$= A_1 [A_1^{*/} C_1^{-1} A_1 + C_2^{*-1} - C_2^{*-1} A_2 (A_2^{*/} C_2^{*-1} A_2) A_2^* C_2^{*-1}]^{-1} A_1^* C_1^{-1} \underline{y}_e \qquad (2.18)$$

The proof of this corollary is similar to the proof of theorem (2.2) and is left to the readers.

FORECASTING OF A SET OF FUTURE PRIVATE INVESTIMENTS AND PRIVATE CONSUMPTIONS OF PAKISTAN

The individual firm works within an industrial national economic and international economic setting and these form broad parameters to its growth and prosperity. The firm has only a minor influence on these factors, so it is very much a case of the forecaster making his forecasts of the firm fit in with these parameters. Thus in preparing a forecast of, say, a firm's growth in the next five or ten years, the forecaster has to acknowledge and make forecast of the general growth expected by the particular industry or the particular market and the growth in the national economy as a whole. This can be done explicitly or semi-implicitly, with the forecaster making subjective assessment based on the macro-forecasts of the national economy from the published such data. Despite the difficulties for forecasting the uncontrollable environment, especially that relating to the actions of government and their economic impact, it is necessary if useful forecasts of the firm are to be obtained, the forecaster must ensure that the forecast of the firm are at least consistent with the macro-forecast.

In this paper the attempt has been made to forecast the private consumptions and private investments of Pakistan to facilitate the forecaster for the forecasting of the desired characteristics / variables of firms at micro-level. Cohen *et al*. [1985] in building a simple Inter-Industry model of Pakistan have forecasted the private consumptions and private investments through a behavioral relationship by ordinary least square method and as a factor of production which increased the capacity for production through a technical relationship. However, the forecasted values obtained in this paper are very poor and the deviations from the actual values are very large. In the present study the results derived in section 2 are applied (because in the behavioral relationship of these variables the second stage prior distribution can be formulated).

For the forecast of private consumption and investment

Let $COP \sim N(\beta_0 + \beta_1 .GDP + \beta_2.REM , C_1)$

Where COP, GDP and REM denotes the private consummations, total gross value of domestic product at factor costs and remittances respectively, and $\underline{\theta}_1$, used in section 2 is given by $\underline{\theta}_1 = (\beta_0, \beta_1, \beta_2)$ where β_1 and β_2 are the rates of change in consumption with respect to GDP and Remittance respectively. We perceive that these rates of change may be influenced by rate of interest, per capita income and rate of inflation etc. i.e. $\underline{\theta}_1 \sim N(A_2\, \underline{\theta}_2, C_2)$

where, the elements of A_2 are rate of interest, rate of inflation and per capita income.

Similarly $INP \sim N(\gamma_0 + \gamma_1.\ INP_{-1} + \gamma_2.\ GDP, C_1)$ where INP denotes the private investment and γ_1, γ_2 are rates of change in private investment with respect to its own value of one preceding period and GDP respectively.

Again $\underline{\theta}_1$ of section 2 in this case is given by

$$\underline{\theta}_1 = (\gamma_0, \gamma_1, \gamma_2).$$

These rates of change are influenced by growth rate, employment rate, inflation rate, rate of interest and per capita income etc. However for the sake of simplicity and the availability of the quantitative data these rates of change have been expressed in terms of inflation rate, interest rate and per capita income only. The data for these exogenous and indigenous variables have been taken from Economic Survey of Pakistan [1986-87] and are detailed in Table 3.1.

Table 3.1: Private Consumptions and Investments (at constant prices of 1959-60)

Year	Private Investment (Million Rs)	Private Consumption (Million Rs)	Gross Domestic Product (Million Rs)	Remittances (Million Rs)	Rate of Interest	Rate of Inflations	Per Capita Income
1959-60	800.00	14,575	16,826	32	2.39	----	386
1960-61	1,474.68	15,823	17,469	29	3.63	3.97	398
1961-62	1,385.28	16,477	18,710	35	3.74	-1.62	411
1962-63	1,377.25	17,034	20,056	36	3.10	-0.13	429
1963-64	1,665.09	17,936	21,356	103	3.31	5.17	447
1964-65	1,811.62	20,148	23,460	115	4.70	4.40	475
1965-66	2,549.75	19,818	25,126	186	5.12	2.78	497
1966-67	2,575.63	21,894	25,901	161	5.57	9.25	504
1967-68	2,573.05	22,808	27,659	230	6.71	2.03	518
1968-69	2,328.89	24,508	29,454	389	5.65	0.36	541
1969-70	2,531.19	27,329	32,336	379	5.37	3.95	583
1970-71	2,495.69	27,471	32,434	233	6.40	5.12	568
1971-72	2,467.57	27,178	32,812	306	5.57	6.34	556
1972-73	2,472.66	28,942	35,179	894	5.68	15.33	585
1973-74	1,736.77	32,863	37,901	703	8.48	22.81	621
1974-75	1,623.38	34,528	39,393	1004	10.63	25.16	611
1975-76	1,606.12	34,918	40,699	1164	9.15	12.31	624
1976-77	1,759.32	36,255	41,727	1783	10.03	9.05	638
1977-78	1,844.58	39,559	44,630	3404	11.20	7.88	686
1978-79	1,847.26	42,034	47,762	3891	8.99	7.16	695

Source: Economic Survey of Pakistan Statistical Supplement 1986-87

The forecasted values of the private consumptions and private investments for the period 1981-82 to 1987-88 are computed from (2.18). All the calculations are made with the help of EXEL. The average forecasts and their 95% predictive intervals along with the available actual values are detailed in Table 3.2.

CONCLUSIONS

This paper reported on the forecasting model based on the Lindley and Smith's M-Stage model and on the basis of relevant data for the period from 1959-60 to 1978-79. We forecasted the Private Consumptions and

Investments by the use of model developed in sections 2 and 3. The degree of reliability can be gathered from a comparison between the solution and observed values of 1981-82 to 1987-88. These results showed the consistency in the projected results as compared to the similar forecasts obtained in Cohen *et al.* [1985]. The mean squared error (MSE) of the projected values obtained in Cohen *et al.* [1985] and the present model have been computed and it is found that the value of MSE for the present model is 34 percent and 42 percent that of MSE computed from the Cohen et al model for the Consumptions and Investments respectively.

Table 3.2: Forecasted values of private consumptions and investments of Pakistan for the period 1981-82 to 1987-88

Year	Private Consumptions (Million Rs.)			Private Investments (Million Rs.)		
	Actual	Forecasted Values		Actual	Forecasted Values	
		OLS	Bayesian		OLS	Bayesian
1981-82	51,028	47,949	50,743	2,805	2,567	2,782
1982-83	53,526	55,935	54,097	3,359	2,842	3,256
1983-84	57,182	59,217	57,761	3,719	3,040	3,696
1984-85	62,837	62,714	61,934	4,070	3,198	3,979
1985-86	67,544	66,464	66,957	4,310	3,338	4,328
1986-87	71,019	70,466	70,284	4.834	3,472	4,812
1987-88	74,856	74,745	75,971	4,920	3,606	5,227

This model promised, among other things to provide a systematic analytical Bayesian framework for forecasting the demands, sales, inventories and costs etc. and for assessing the relative strengths of the many forces that molded this structure over time. Despite the imperfection of this model, in terms of specification, which we admit, the results of the model are statistically significant.

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